YOU WANT PROOF?

I'LL GIVE YOU PROOF!

How To Write Math Proofs

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Objectives

- Direct Proofs
- Proof by Contradiction
- Proof by Contrapositive
- If, and only if
- Proof by Mathematical Induction
- Real World Applications

Direct Proofs

- Characterized by being simple and short; direct proofs can be done by using relatively basic techniques.

Proof that Divisibility is Transitive

- If a and b are two natural numbers, we say that a divides b if there is another natural number k such that $b = a\,k$. For example, 8 divides 24 because there is a natural number k (namely 3) such that $24 = 8\,k$

- Theorem: If a divides b and b divides c , then a divides c and b divides c

- Proof: By our assumptions, and the definition of divisibility, there are natural numbers k_1 and k_2 such that $b = a\,k_1$ and $c = b\,k_2$. Consequently, $c = b\,k_2 = a\,k_1\,k_2$ and let $k = k_1\,k_2$. Now, k is a natural number and $c = a\,k$, ergo by the definition of divisibility, a divides c

Proof by Contradiction

- When one states the opposite of what one wants to prove, prove the opposite is not possible.

Infinitely Many Primes

- Assume for contradiction that there is a finite number of primes. Thus, the set of primes can be represented as $\{a, b, c, \ldots n\}$. The product of this set plus one, $a \cdot b \cdot c \ldots n + 1$ represented by q, must be either prime or composite. Since each prime isn't divisible by anything else, and q can't be divisible by any a, q must be prime. Since q is a prime number not included in the set of all primes, it contradicts the assumption that all primes are in the list $\{a, b, c, \ldots n\}$

Proof by Contrapositive

- Logically, the statements " p implies q " and "Not p implies not q " are equivalent
- For example, the contrapositive of the statement "If it's legal, then it's fresh" is "if it isn't fresh, it isn't legal" (please don't sue us Legal Sea Foods)
- This can be used in proofs by showing the contrapositive of a theorem is true, thus the theorem itself if true

Logic: another thing that penguins aren't very good at.

Parity

- A number's parity describes whether it is even or odd; the parity of 3 is odd, the parity of 4 is even
- Even is defined as a number which can be represented by $2k$, where k is an integer. Odd is defined as a number which can be represented by $2k+1$, where k is also an integer
- Theorem: if x and y are two integers for which $x+y$ is even, then x and y have the same parity
- Proof: the contrapositive version of this theorem is "If x and y are two integers with opposite parity, then their sum must be odd." So we assume x is even and y is odd. Thus there are integers k and m for which $x = 2k$ and $y = 2\mathrm{m} + 1$. Now then, we compute the sum $x + y = 2k + 2m + 1 = 2(k+m) + 1$ which is an odd integer by definition because $k + m$ must be an integer

If, and only if a.k.a iff

- Indicative of a proof that is reversible.
- It requires the ability to show that the first implies the second and vice versa.

Examples

- A person is a bachelor iff that person is a marriageable man who has never married.
- Cheese is good iff it is from Europe.
- A person is great iff they are us.

Proof that "All Girls are Evil"

First we state that girls require time and money.

$$\text{Girls} = \text{Time} \times \text{Money}$$

And as we all know "time is money."

$$\text{Time} = \text{Money}$$

Therefore:

$$\text{Girls} = \text{Money} \times \text{Money} = (\text{Money})^2$$

And because "money is the root of all evil":

$$\text{Money} = \sqrt{\text{Evil}}$$

Therefore:

$$\text{Girls} = \left(\sqrt{\text{Evil}}\right)^2$$

And we are forced to conclude that:

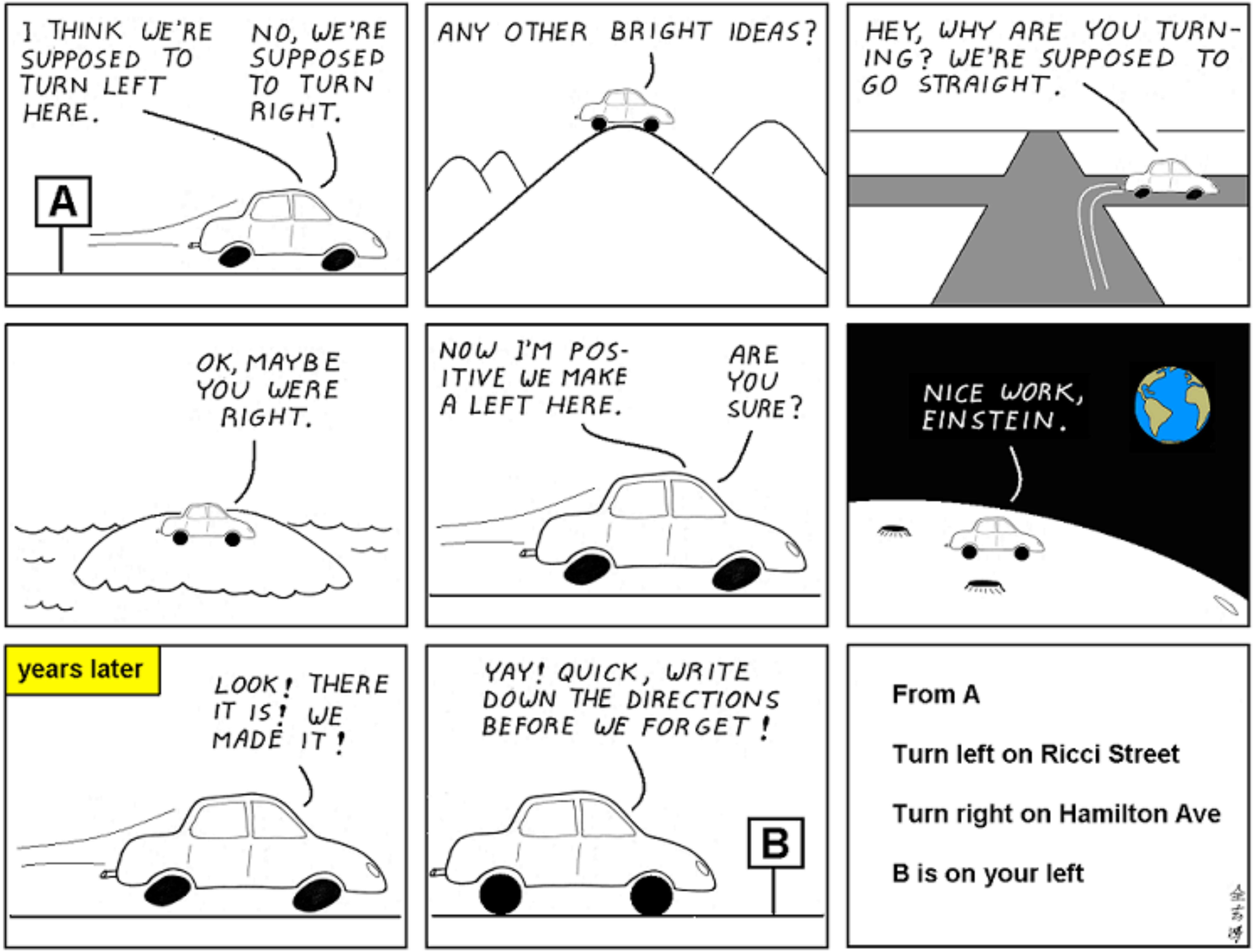
$$\text{Girls} = \text{Evil}$$

If a = b (so I say) [a = b]
And we multiply both sides by a
Then we'll see that a2 [a2 = ab]
When with ab compared
Are the same. Remove b2. OK? [a2− b2 = ab − b2]

Both sides we will factorize. See?
Now each side contains a − b. [(a+b)(a − b) = b(a − b)]
We'll divide through by a
Minus b and olé
a + b = b. Oh whoopee! [a + b = b]

But since I said a = b
b + b = b you'll agree? [b + b = b]
So if b = 1
Then this sum I have done [1 + 1 = 1]
Proves that 2 = 1. Q.E.D.

If you think that this proof is a hit
And you're enamored with your clever wit
Then look close and you'll see
That in part two, line three,
You divided by zero - OH SH-



This is how most mathematical proofs are written.

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