Logical Implication

Good news: you already know a lot of this implicitly. For example, fill in the blanks:

Either Joe went to the movie, or he stayed home.
Joe didn't stay home.
Therefore, ______ .

If you don't take an umbrella, you'll get wet.
You didn't take an umbrella.
Therefore, ______ .

If Kieran comes to the party but Pierre doesn't, Nora will be sad.
Nora wasn't sad.
Therefore, ______ .

When we say that some premises **logically imply** a conclusion, what we mean is: there's *no way* to make the premises true and the conclusion false. If the premises logically imply the conclusion, the argument is **logically valid**.

- When you filled in the blanks above, you were completing valid arguments.

If we can think of a possible situation that makes the premises true and the conclusion false, that's a **counterexample**. It shows that the premises don't logically imply the conclusion, which means the argument is logically invalid.

If an argument is valid *and* has true premises, then it's **sound**.

- Sometimes it's hard to tell whether the premises are true. But a valid argument tells you that the conclusion is *at least* as plausible as the premises.

Practice

Say whether each of the arguments below is logically valid. If it's not, come up with a counterexample.

i) Ducks are mammals.
Mammals lay eggs.
Ducks lay eggs.

ii) If Suzy is a baseball player, then she has strong arms.
Suzy has strong arms.
Suzy is a baseball player.

iii) Any space alien that speaks a human language either has an electronic translator or has vocal chords.
E.T. is a space alien who doesn't have an electronic translator.
E.T. has vocal chords.

iv) If lions and tigers live in the jungle, then so do monkeys.
Monkeys don't live in the jungle.
Lions don't live in the jungle.

v) If Livi has jellybeans, then Skyler will steal them and eat them.
Livi has jellybeans, but Skyler won't steal them.
The moon is made of green cheese.

Truth Tables

Looking for a counterexample isn't always the most efficient way to check for validity. You can't always be sure there isn't one. But you *can* be sure if you make a truth table!

Example:

Either Andy is the murderer and Boris didn't steal the diamonds, or Carrie didn't disable the alarm. If Boris stole the diamonds, then Carrie disabled the alarm. So if Boris stole the diamonds, then Andy is the murderer.

To check this for validity, we have to check all the situations that make the premises true, and see whether they all make the conclusion true. We only need to care about situations that affect the truth of these sentences:
"Andy is the murderer" = A
"Boris stole the diamonds" = B
"Carrie disabled the alarm" = C

Why are A, B, and C important?
- We care about the things that have truth values (things that can be true or false), since logical validity is about truth and falsity.
- But we don't care (for now!) whether A, B, and C are actually true. So we don't need to know what those sentences mean.
- We want to find the *logical structure* of the premises and conclusion. The smaller sentences A, B, and C don't have logical structure that's important for evaluating this argument, so we don't break them down into smaller parts.

Logical Structure
- Like a map or a recipe. Logical structure shows you how the important parts of a sentence are connected together, while leaving out the irrelevant details.
- The important parts of a sentence are the parts that determine whether it's true or false. Logical structure tells you how they do that.
- Using the logical structure of a sentence, you can figure out what follows from it and how it functions in an argument.

We're working our way down to the logical structure of each big sentence. So far, the argument is:

Either A and not B, or not C.
If B, then C.
If B, then A.

Besides A, B, and C, there are some other words hanging around: 'and', 'not', 'or', etc. these are *connectives*. You can think of connectives as functions (like + and ×).

- A connective takes sentences as input and gives you a new sentence with a truth value that depends on the truth values of the sentences you gave it.
- Each connective depends on its inputs in a different way. We can say how each connective works using a *truth table*.

Here's the truth table for 'and':

P	**Q**	**P & Q**
T	T	T
T	F	F
F	T	F
F	F	F

There are four possible combinations of truth values for P and Q. So we list each one and then say what truth value P&Q has for each combination.

There are five connectives we use a lot in logic. Here are the other four:
(∨ = "or"; → = "if..., then..."; ↔ = "if and only if"; ¬ = "not")

P	**Q**	**P ∨ Q**
T	T	
T	F	
F	T	
F	F	

P	**Q**	**P → Q**
T	T	
T	F	
F	T	
F	F	

P	**Q**	**P ↔ Q**
T	T	
T	F	
F	T	
F	F	

P	**¬P**
T	
F	

Replacing the english words in our argument with their logical symbols, we get this:

$$
\begin{array}{l}
(A \,\&\, \neg B) \lor \neg C \\
\underline{B \to C \qquad\qquad} \\
B \to A
\end{array}
$$

Now we can finally see if the argument is valid! Here's how:

- Make a truth table showing all possible combinations of truth values of A, B, and C (see next page).
- Put the premises and conclusion of the argument in the truth table too.
- Use the truth tables for the connectives to get the truth values of the premises and the conclusion from the truth values of A, B, and C in each possible scenario.
- Look at each possibility that makes the premises true. If all of those make the conclusion true too, then this is a valid argument.

A	B	C	(A & ¬B) ∨ ¬C	B → C	B → A
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

More Practice:

Use truth tables to check whether each of these is valid or not.

1. (A ∨ B) & C
   ¬B → ¬A
   ―――――――
   B & C

2. B ↔ (C → A)
   A → ¬C
   ―――――――
   B

3. (C ∨ A) → (B → A)
   B
   ―――――――
   ¬C ∨ A

4. B & (C ↔ A)
   A ↔ ¬B
   ―――――――
   ¬C & B

5. A ∨ (¬B → C)
   (¬A ∨ ¬C) → B
   ―――――――
   A ↔ B

6. ¬C → (A ∨ B)
   ¬B
   ―――――――
   ¬A & C

7. (A ∨ B) ↔ C
   B ∨ ¬C
   ―――――――
   A ∨ ¬C

8. (B & (C → A)) → ¬C
   B & A
   ―――――――
   B & ¬C