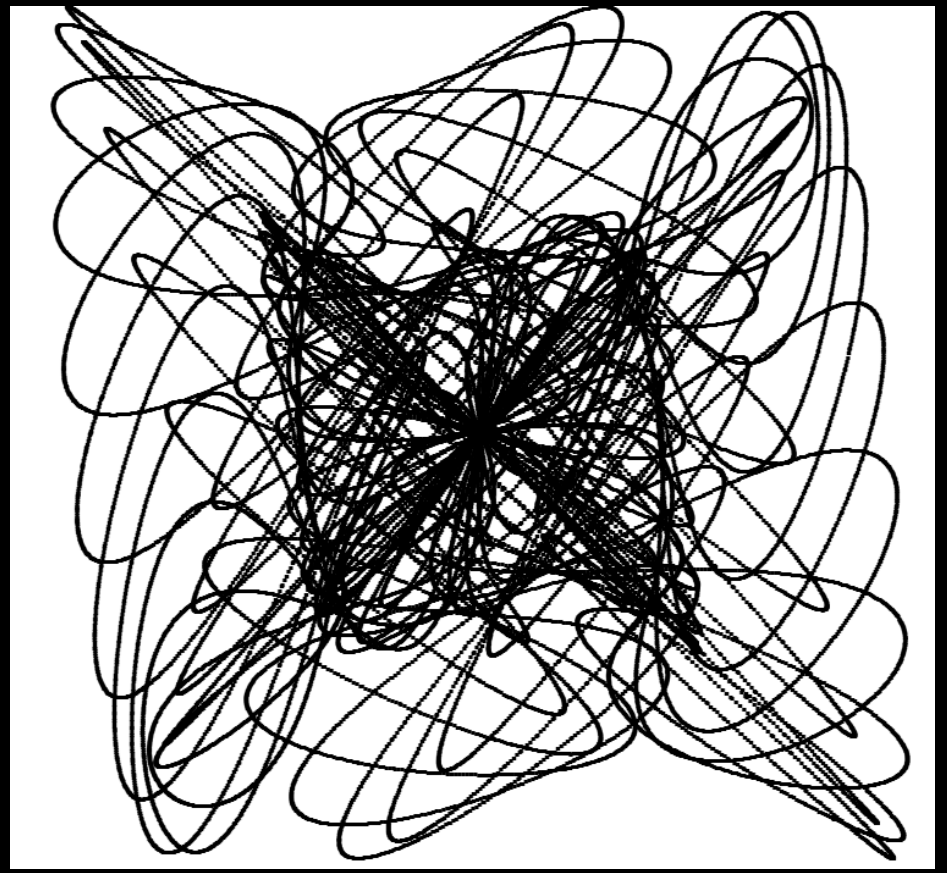
Classical Mechanics



Questions!

- Why are the slaves interested in science for practical purposes?
- Are we going to be learning classic physics?
- What is my second favorite breed of dog?
- What is my opinion on cats?

Comments

- Focus on important information
- Be more interactive!

Summary of Last Lecture

- Ancient Greece
  - Aristotle is major person on matter, motion, and astronomy
  - Concept of beauty and aesthetics in physics and math
  - Ptolemy and epicycles - accurate
  - Lack of practicality – social structure
- Medieval era
  - Science is significantly slowed down, but technical revolution
  - Arab transmission
  - "prescience"
  - Early attacks on Aristotle – shift from $F = Rv$ to $F = mv$

Outline for this Lecture

- Some philosophy stuff
- Copernicus
- Galileo
- Newton
- Lagrange and Hamilton (a little bit)

Pineapple on Pizza!

- MIT Brain and Cognitive Science

To the Reader

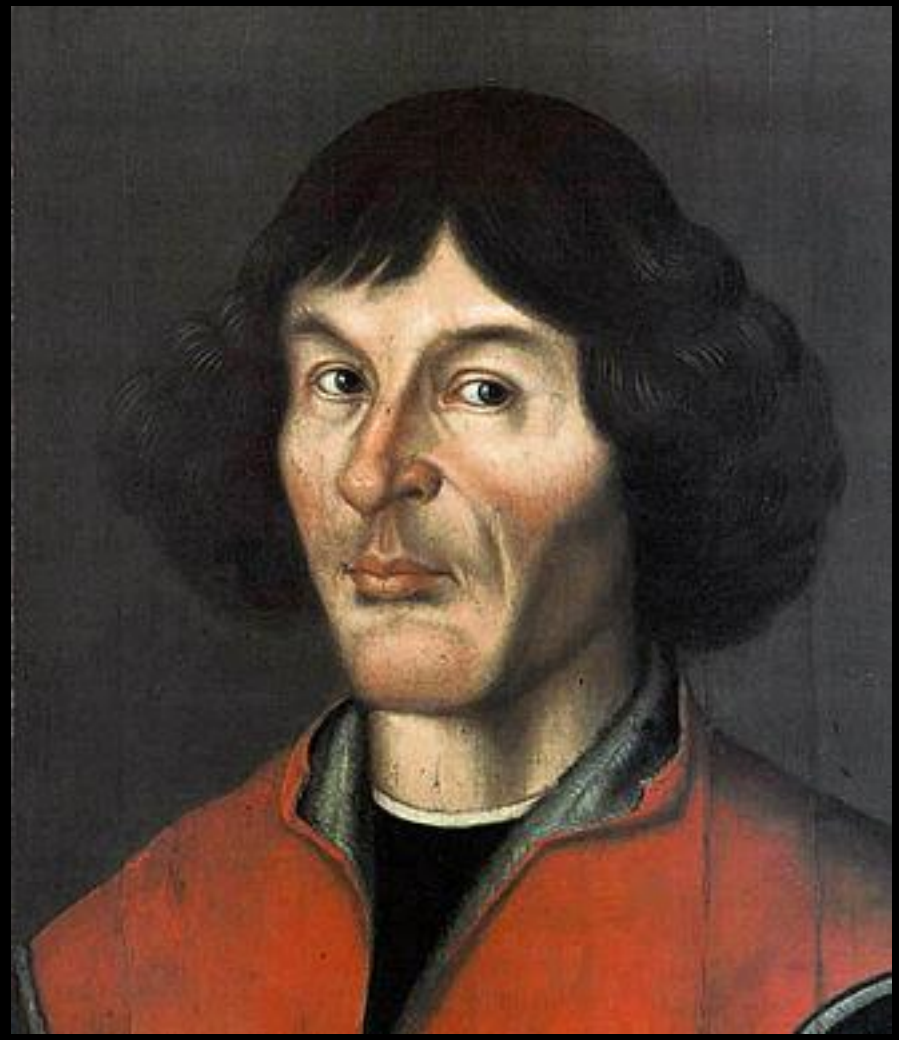
Concerning the Hypotheses of this Work

There have already been widespread reports about the novel hypotheses of this work, which declares that the earth moves whereas the sun is at rest in the center of the universeHence certain scholars, I have no doubt, are deeply offended and believe that the liberal arts, which were established long ago on a sound basis, should not be thrown into confusion. But if these men are willing to examine the matter closely, they will find that the author of this work has done nothing blameworthy. For it is the duty of an astronomer to compose the history of the celestial motions through careful and expert study. Then he must conceive and devise the causes of these motions or hypotheses about them. Since he cannot in any way attain to the true causes, he will adopt whatever suppositions enable the motions to be computed correctly from the principles of geometry for the future as well as for the past. The present author has performed both these duties excellently. For these hypotheses need not be true nor even probable. On the contrary, if they provide a calculus consistent with the observations, that alone is enough. Perhaps there is someone who is so ignorant of geometry and optics that he regards the epicyclc of Venus as probable, or thinks that it is the reason why Venus sometimes precedes and sometimes follows the sun by forty degrees and even more. Is there anyone who is not aware that from this assumption it necessarily follows that the diameter of the planet at perigee should appear more than four times, and the body of the planet more than sixteen times, as great as at apogee? Yet this variation is refuted by the experience of every age. In this science there are some other no less important absurdities, which need not be set forth at the moment. For this art, it is quite clear, is completely and absolutely ignorant of the causes of the apparent nonuniform motions. And if any causes are devised by the imagination, as indeed very many are, they are not put forward to convince anyone that are true, but merely to provide a reliable basis for computation. However, since different hypotheses are sometimes offered for one and the same motion (for example, eccentricity and an epicycle for the sun's motion), the astronomer will take as his first choice that hypothesis which is the easiest to grasp. The philosopher will perhaps rather seek the semblance of the truth. But neither of them will understand or state anything certain, unless it has been divinely revealed to him.

Therefore alongside the ancient hypotheses, which are no more probable, let us permit these new hypotheses also to become known, especially since they are admirable as well as simple and bring with them a huge treasure of very skillful observations. So far as hypotheses are concerned, let no one expect anything certain from astronomy, which cannot furnish it, lest he accept as the truth ideas conceived for another purpose, and depart from this study a greater fool than when he entered it. Farewell.

Copernicus (1473 – 1543) – Revolution?

- Two works
  - Commentariolus (1514)
  - De revolutionibus orbium coelestium (1543)
- The Copernican Revolution…
  - Put sun at rest and the Earth among other planets
  - Destroyed sublunary ideas of Aristotle – Earth just like any other planet
  - Created a model to substantiate claims



Copernicus (1473 – 1543) – Revolution?

- Two works
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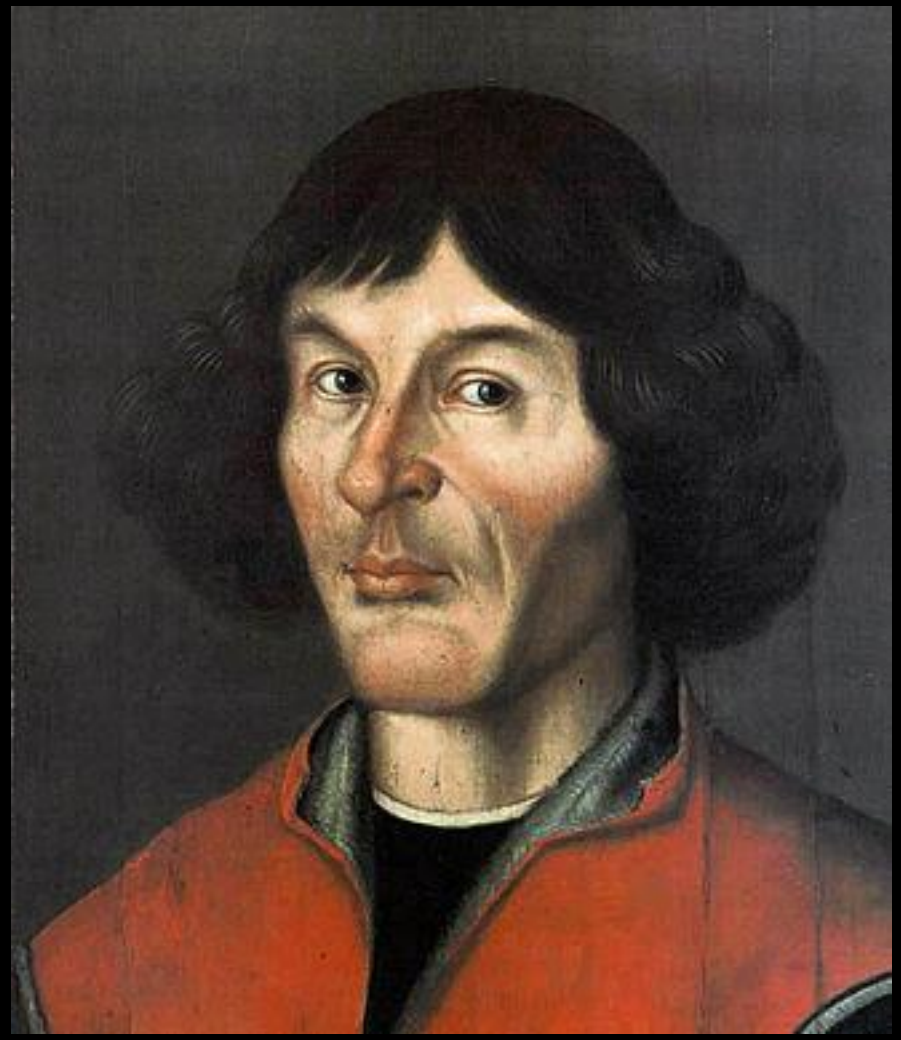
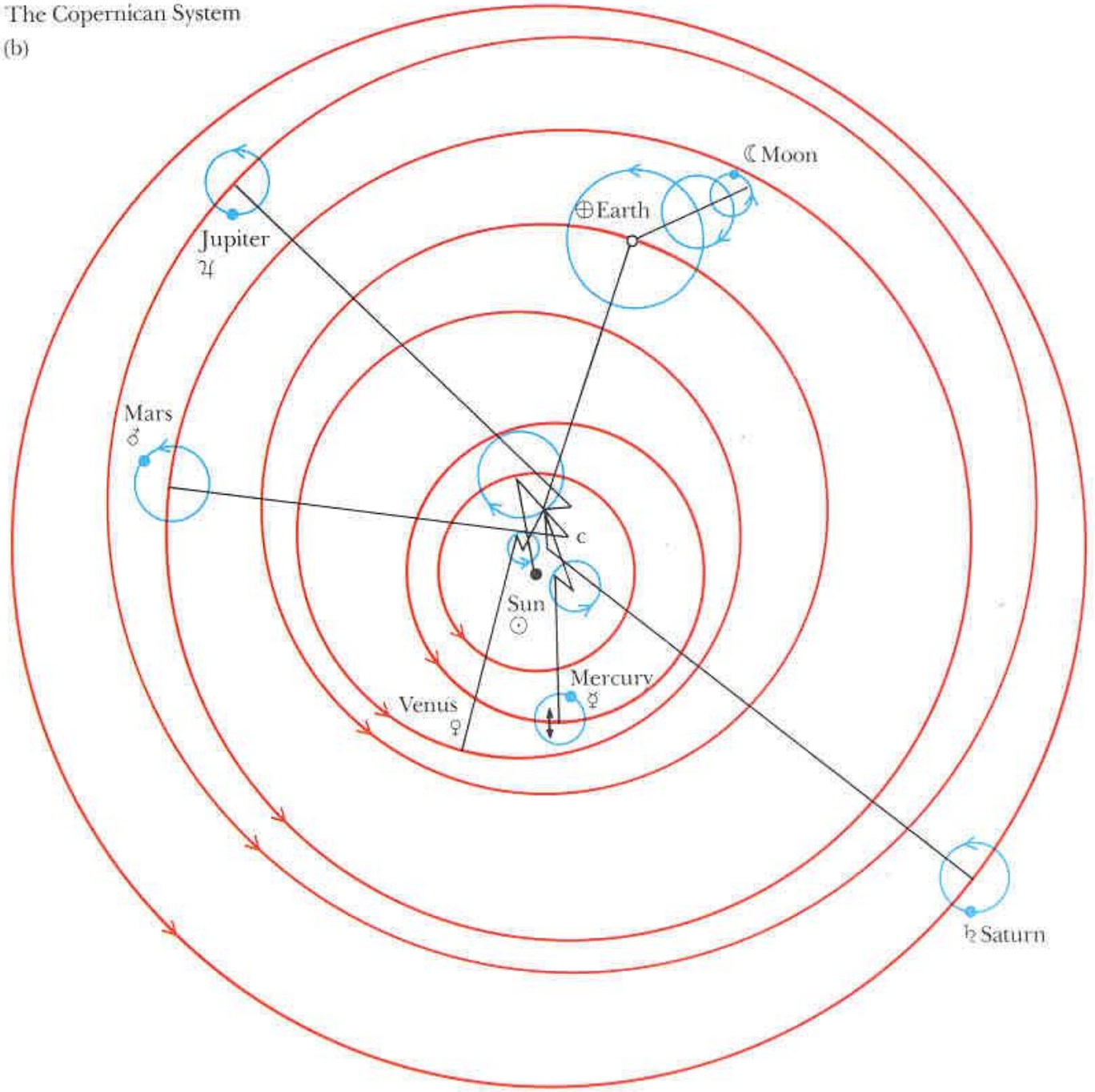
  - De revolutionibus orbium coelestium (1543)
- The Copernican Revolution…
  - Put sun at the center and the Earth among other planets
  - Destroyed sublunary ideas of Aristotle – Earth just like any other planet
  - Created a model to substantiate claims
- And not quite there
  - Double epicycles!!!
  - Circular orbits - aesthetics



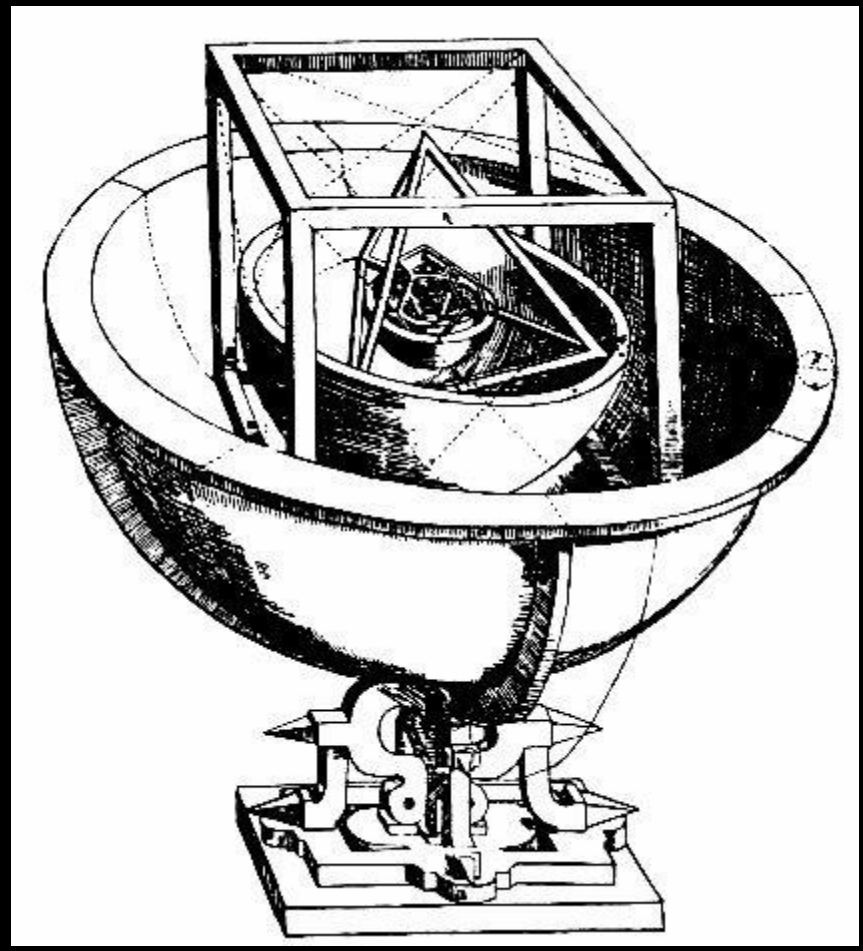
The Copernican System

(b)



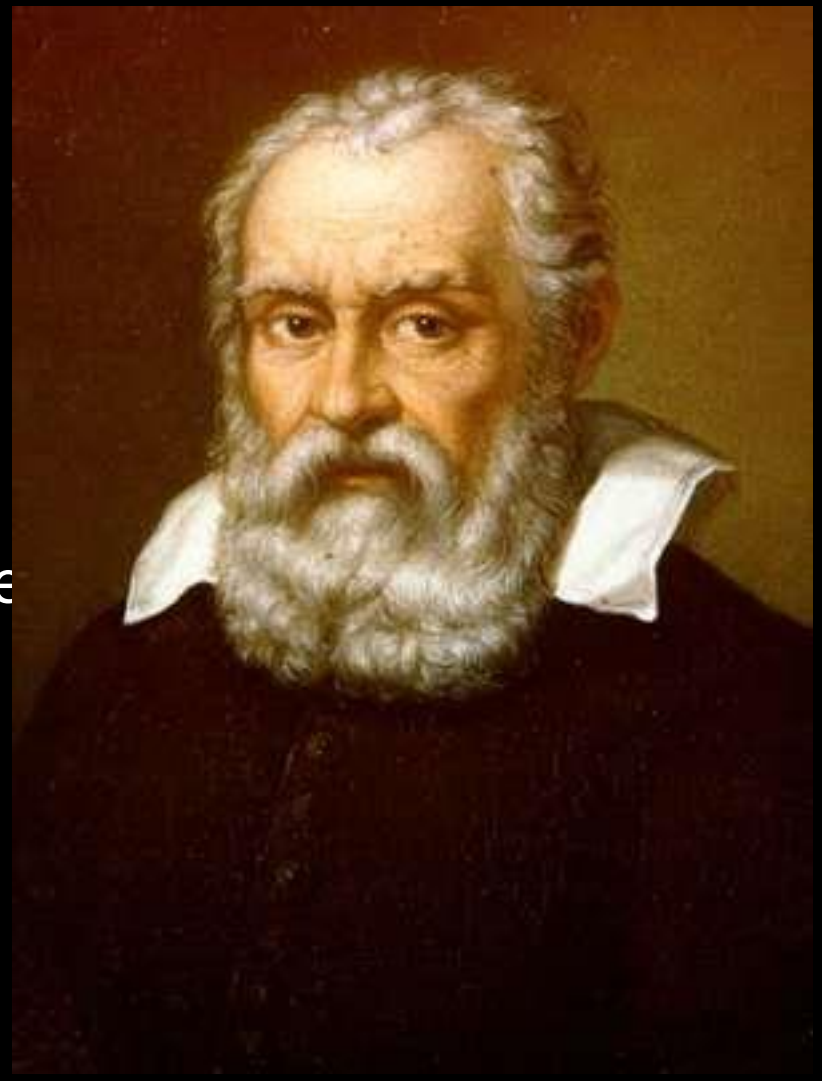
Brahe and Kepler

- Tycho Brahe (1546-1601)
  - Really a step backwards, moved back to geocentrism
  - Advanced astronomical equipment and observational data
  - Nova (1572) and comet (1576)
- Johannes Kepler (1571-1630)
  - Highly interested in mysticism and numerology
  - Kepler's laws – elliptical orbits!



Galileo (1564 – 1642)

- Telescopes
- Starry Messenger (1610)
  - Mountains on the moon
  - Sunspots
  - Jupiter's moons
  - Phases of Venus discovered after publication
- Dialogues Concerning the Two Chief World Syste (1632)
  - Written in Italian
  - Salviati, Sagredo, and Simplicio
  - Debunk Aristotle as much as possible – relativity!
- Two New Sciences (1638)
  - Material strength and kinematics
  - Air resistance and pendulum motion



Kinematics, math, and experimentation

- Constant acceleration, a
  - $v(t) = at$
  - The average velocity is $v_{avg} = \frac{1}{2}at$
  - $d = v_{avg}t = \left(\frac{1}{2}at\right)t = \frac{1}{2}at^2$

Kinematics, math, and experimentation

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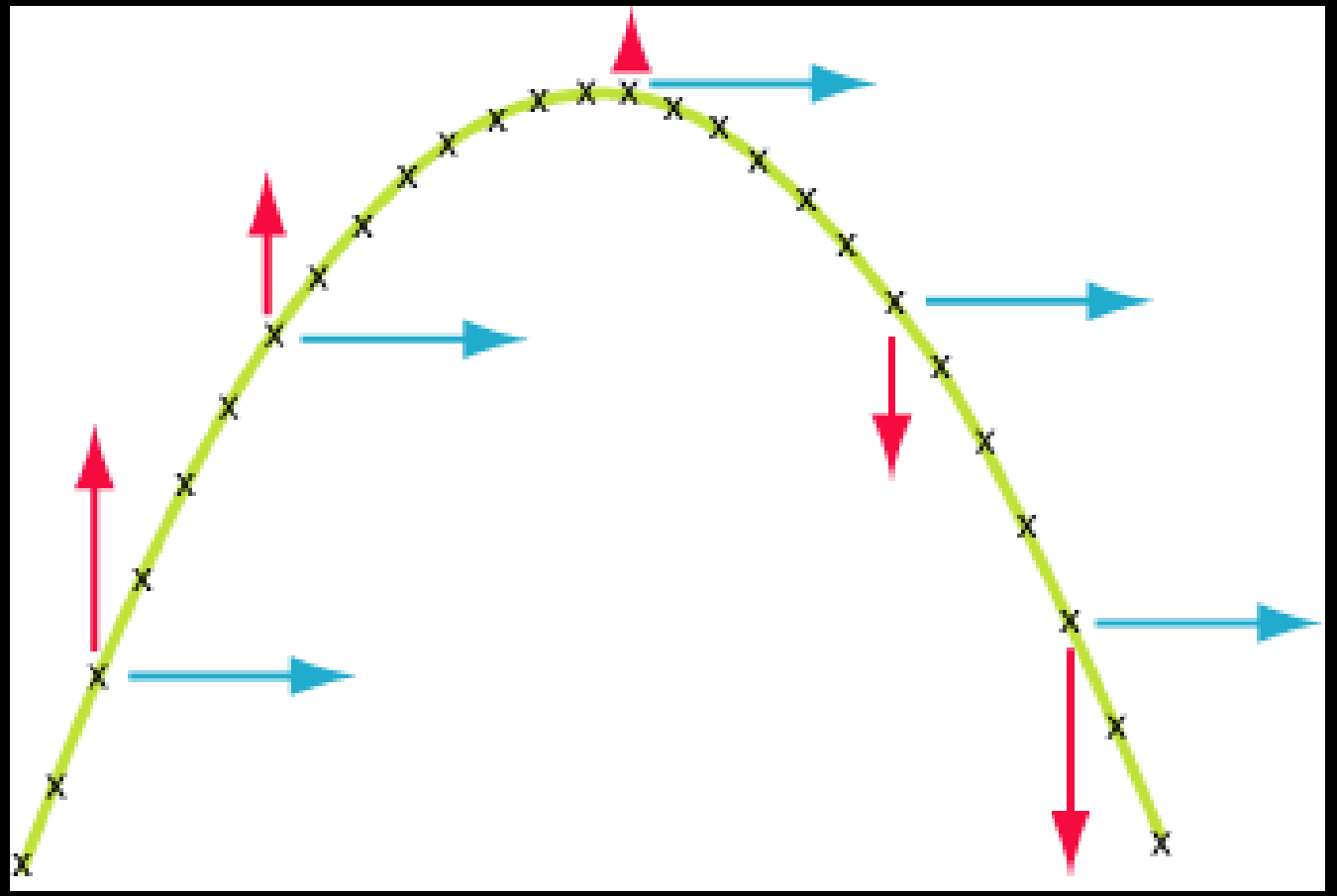
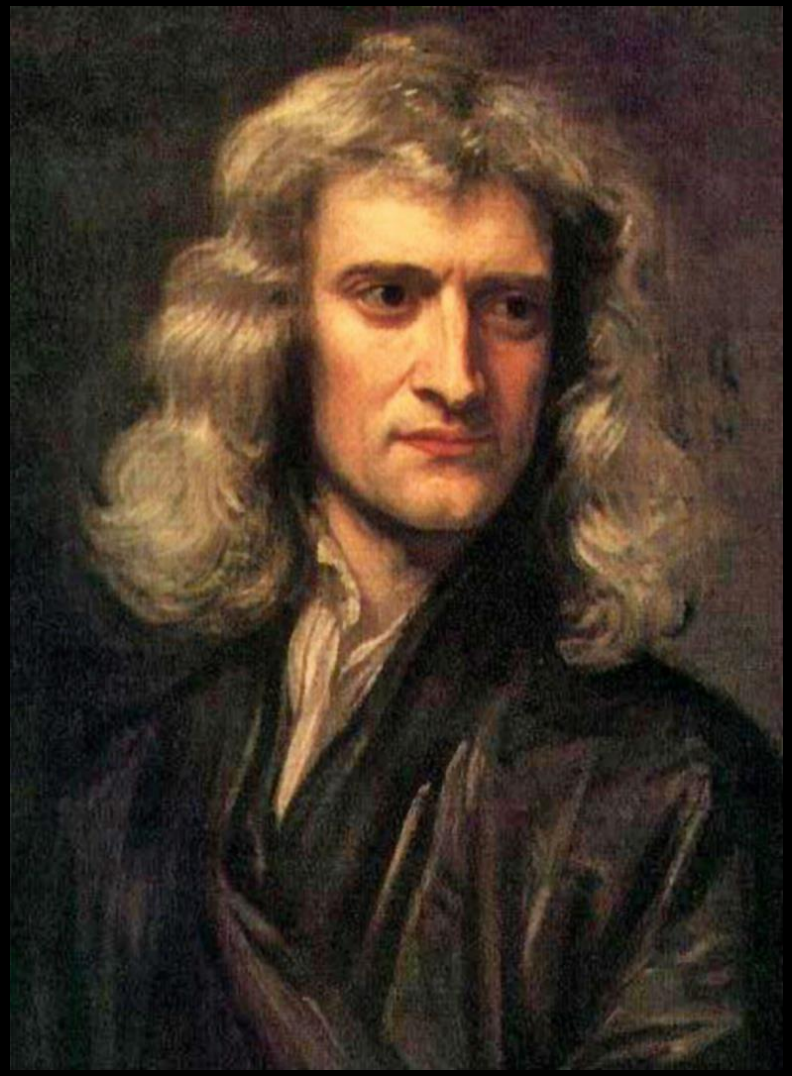
  - $d = v_{avg}t = \left(\frac{1}{2}at\right)t = \frac{1}{2}at^2$
- This is the distance a falling ball travels!!!
- Quadratic relationship – parabola
- How do we prove this?

“Project” in Projectile



Isaac Newton (1643-1727)

- Built on works of Descartes and Huygens
- Plague, alchemy, religion, and the economy
- Newton's Philosophiæ Naturalis Principia Mathematica (1687)
  - Book 1 – Motion in vacuum
  - Book 2 – Motion in media
  - Book 3 – Gravitation – inverse square law
- Opticks (1704)
  - English – easily accessible
- "If I have seen further than others, it is by standing upon the shoulders of giants."



Newton's Laws of Motion

add to notes

Newton's First Law:

Objects in motion tend to stay in motion and objects at rest tend to stay at rest unless acted upon by an unbalanced force.

Newton's Second Law:

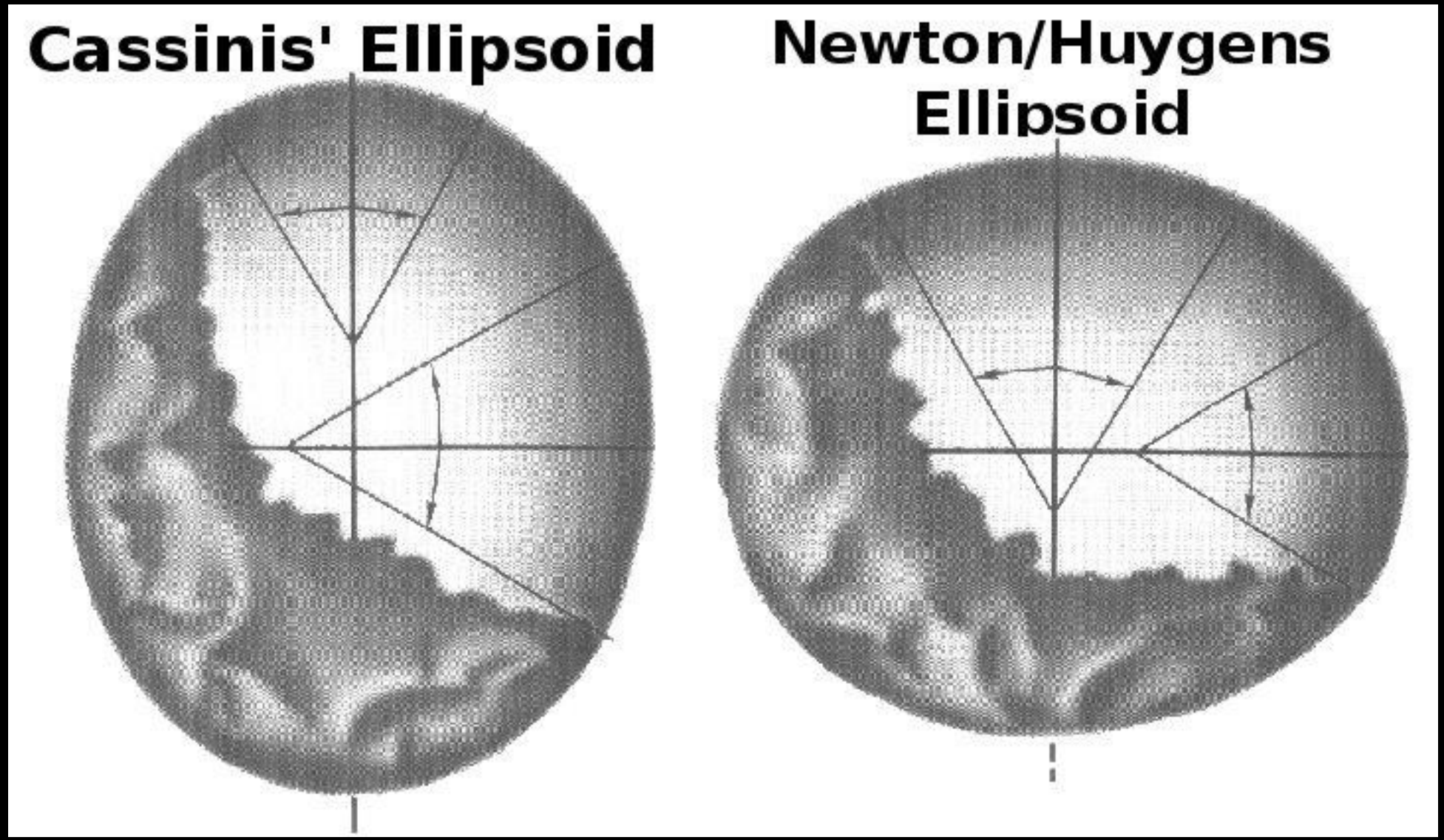
Force equals mass times acceleration (F = ma).

Newton's Third Law:

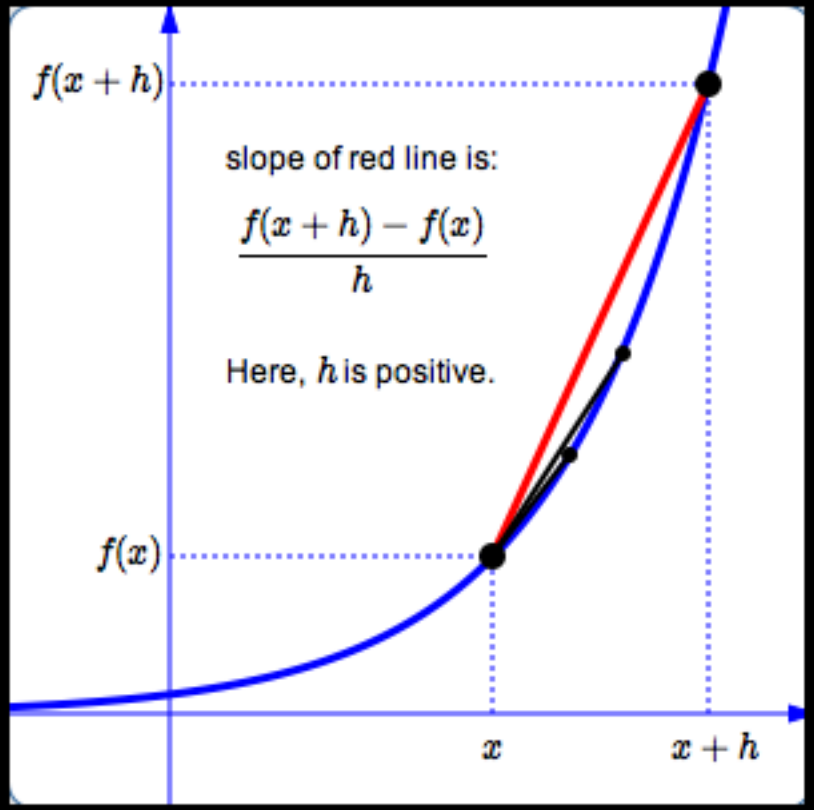
For every action there is an equal and opposite reaction.

Newtonianism and Enlightenment

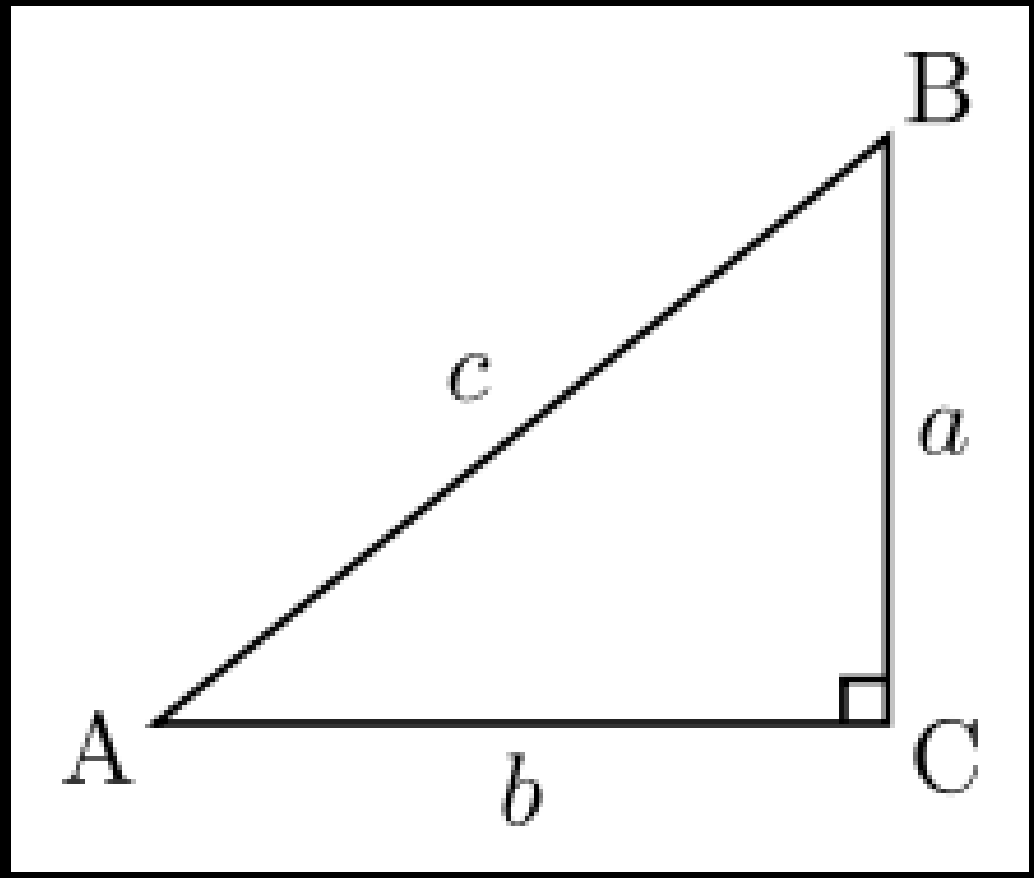
- Newton was a “hero”
  - The image of Newton used for political, economic, cultural gain
  - Rise in patents and entrepreneurship, diving bell
  - Desaguliers – mythbuster
- French society and intellectuals
  - Philosophe and the salon, edgy and rebellious
  - Émilie du Châtelet and Maupertuis


Cassinis' Ellipsoid
Newton/Huygens
Ellipsoid

Review of Algebra/Trigonometry

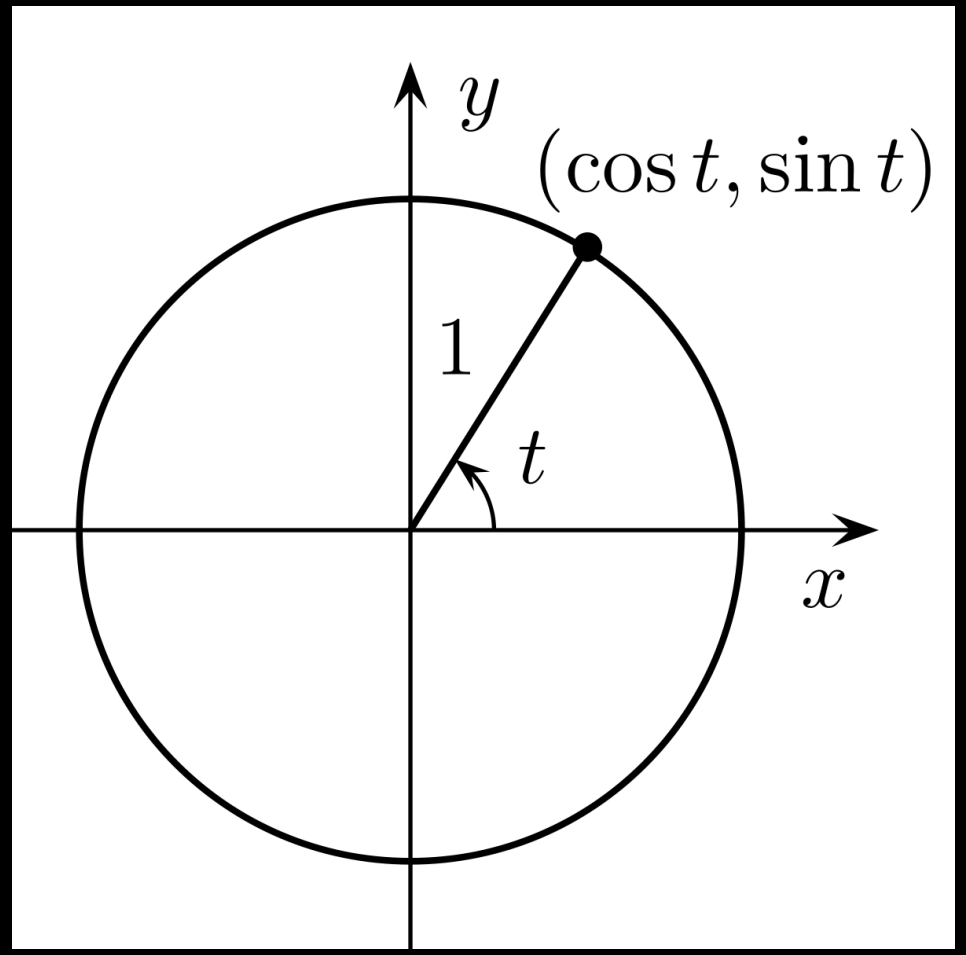


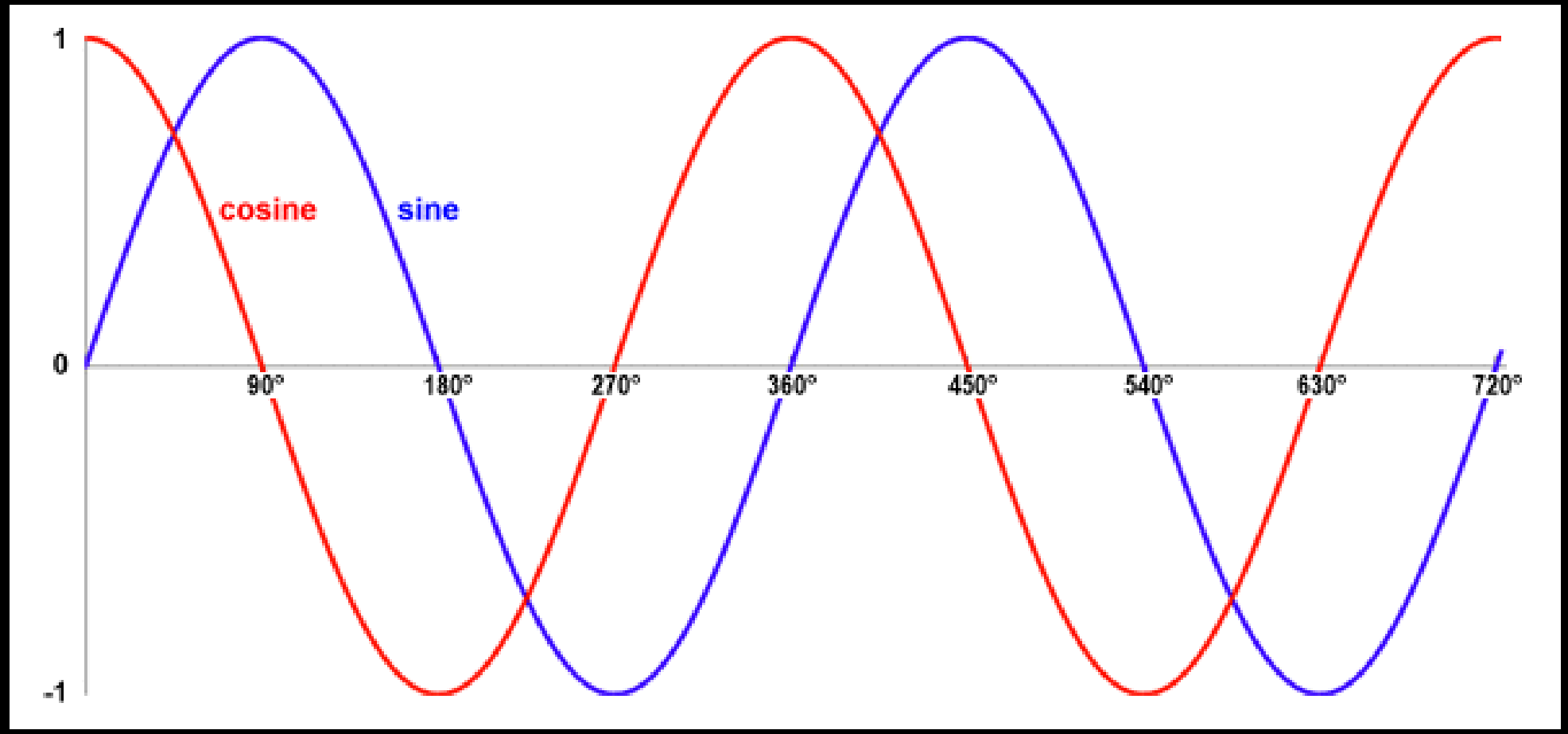
Review of Algebra/Trigonometry



Maybe New Trigonometry?

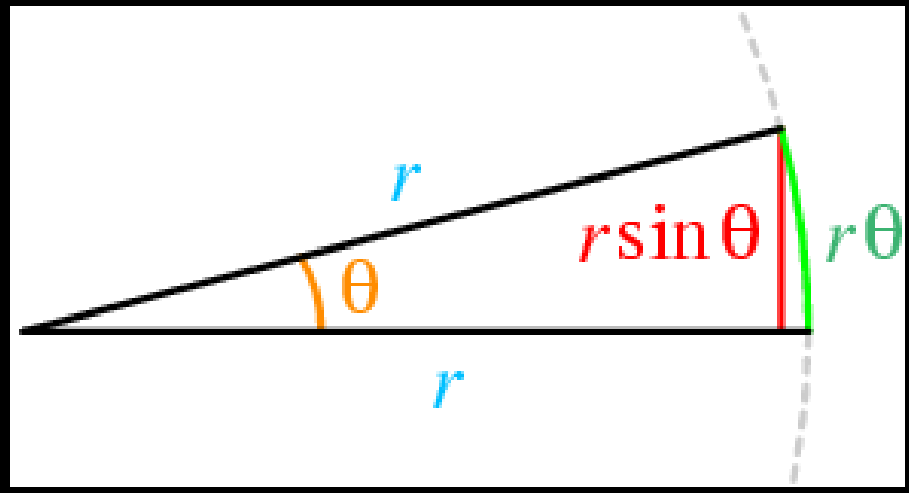
- Extend definition of sine and cosine function to all angles
- Cosine is the x coordinate of a point on a unit circle, sine is the y coordinate
- What is $\cos(135°)$?




1
0
-1
cosine
sine
90°
180°
270°
360°
450°
540°
630°
720°

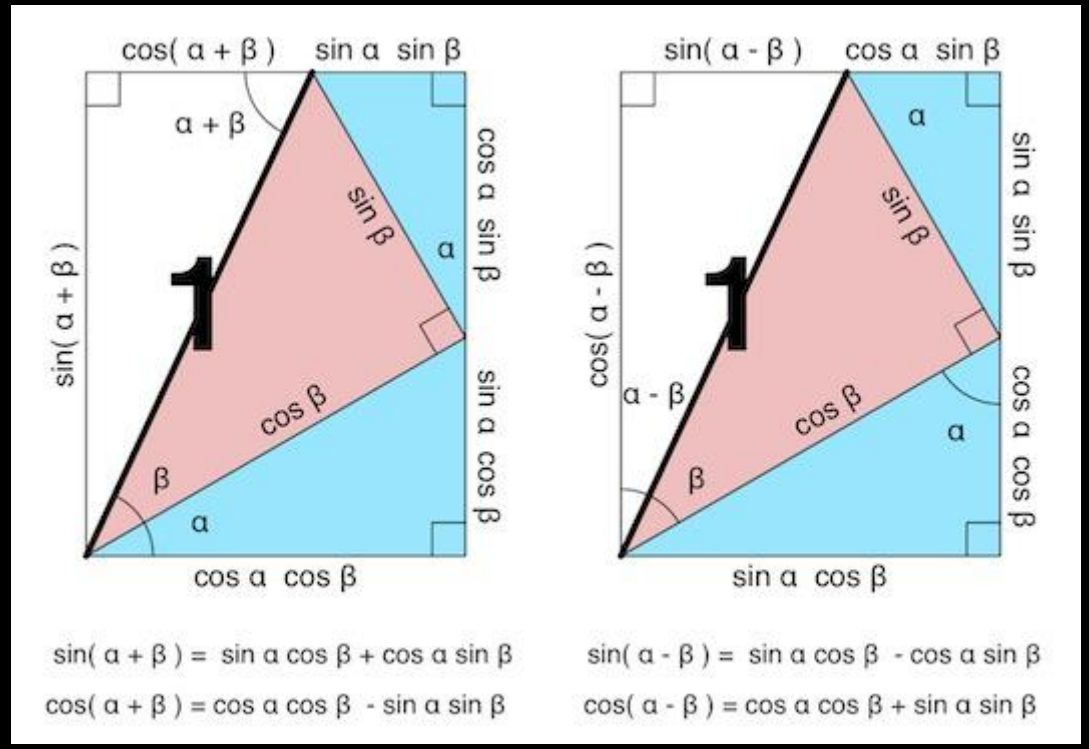
Maybe New Trigonometry?

- If we define angles in terms of radians, we have a convenient formula for the arc length of a circle
- 2π radians $= 360°$
- Thus, the length of an arc subtended by an angle θ is $r\theta$
- Note: For small angles defined in terms of radians, $\sin\theta \approx \theta$



Maybe New Trigonometry?

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$



Calculus!

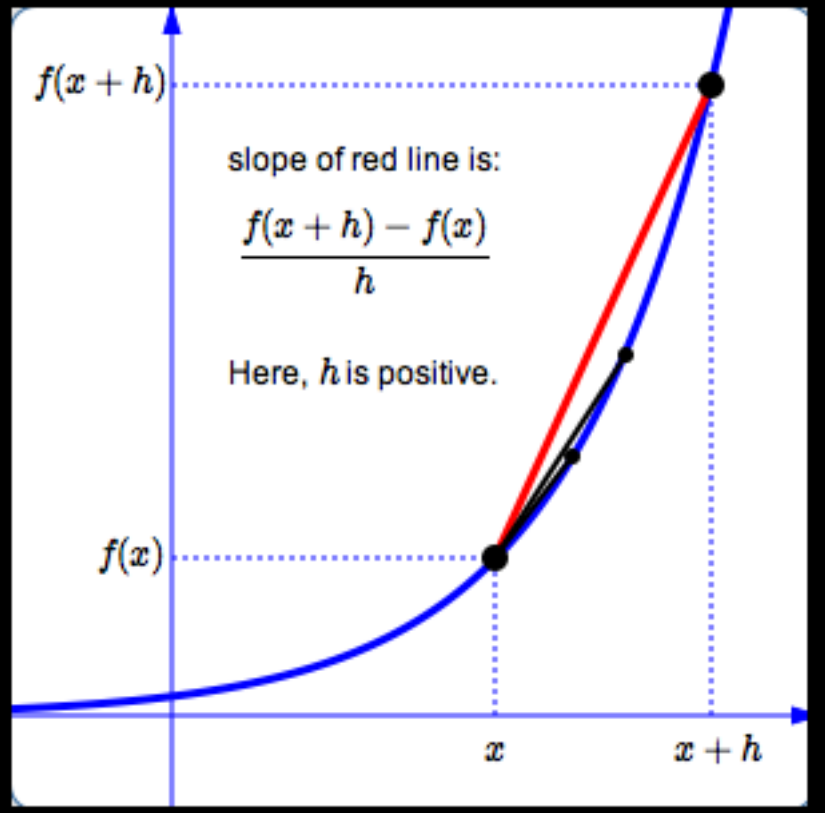
- How do we measure instantaneous velocity?

Calculus!

- How do we measure instantaneous velocity?
- Measure the difference coefficient of the position function and make h very small
- This is the derivative: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
- Also expressed as $\frac{df}{dx}$
- Example: $x(t) = 10t$

$$\rightarrow \frac{dx}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}$$

$$= \lim_{h \to 0} \frac{10(t+h) - 10t}{h} = 10$$



Calculus!

- What is the derivative of $\mathrm{x(t)} = \sin(t)$?
- $\frac{dx}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h} = \lim_{h \to 0} \frac{\sin(t+h) - \sin(t)}{h}$

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- $\sin(t + h) = \sin(t)\cos(h) + \cos(t)\sin(h)$

Calculus!

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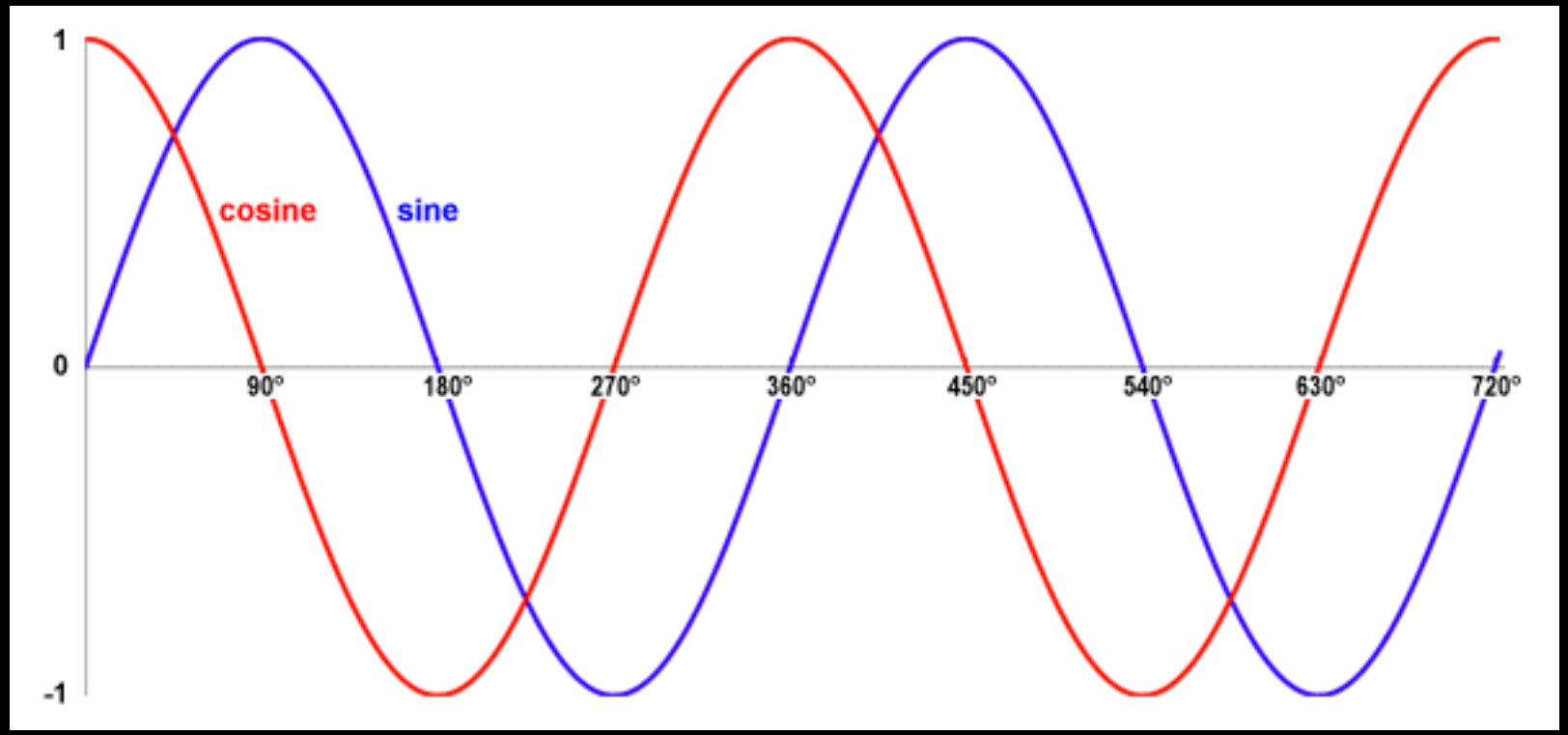
- $\sin(t + h) = \sin(t)\cos(h) + \cos(t)\sin(h)$
- $\lim_{h \to 0} \frac{\sin(t+h) - \sin(t)}{h} = \lim_{h \to 0} \frac{\sin(t)\cos(h) + \cos(t)\sin(h) - \sin(t)}{h}$

$$= \lim_{h \to 0} \frac{\cos(t)\, h}{h} = \cos(\mathrm{t})$$

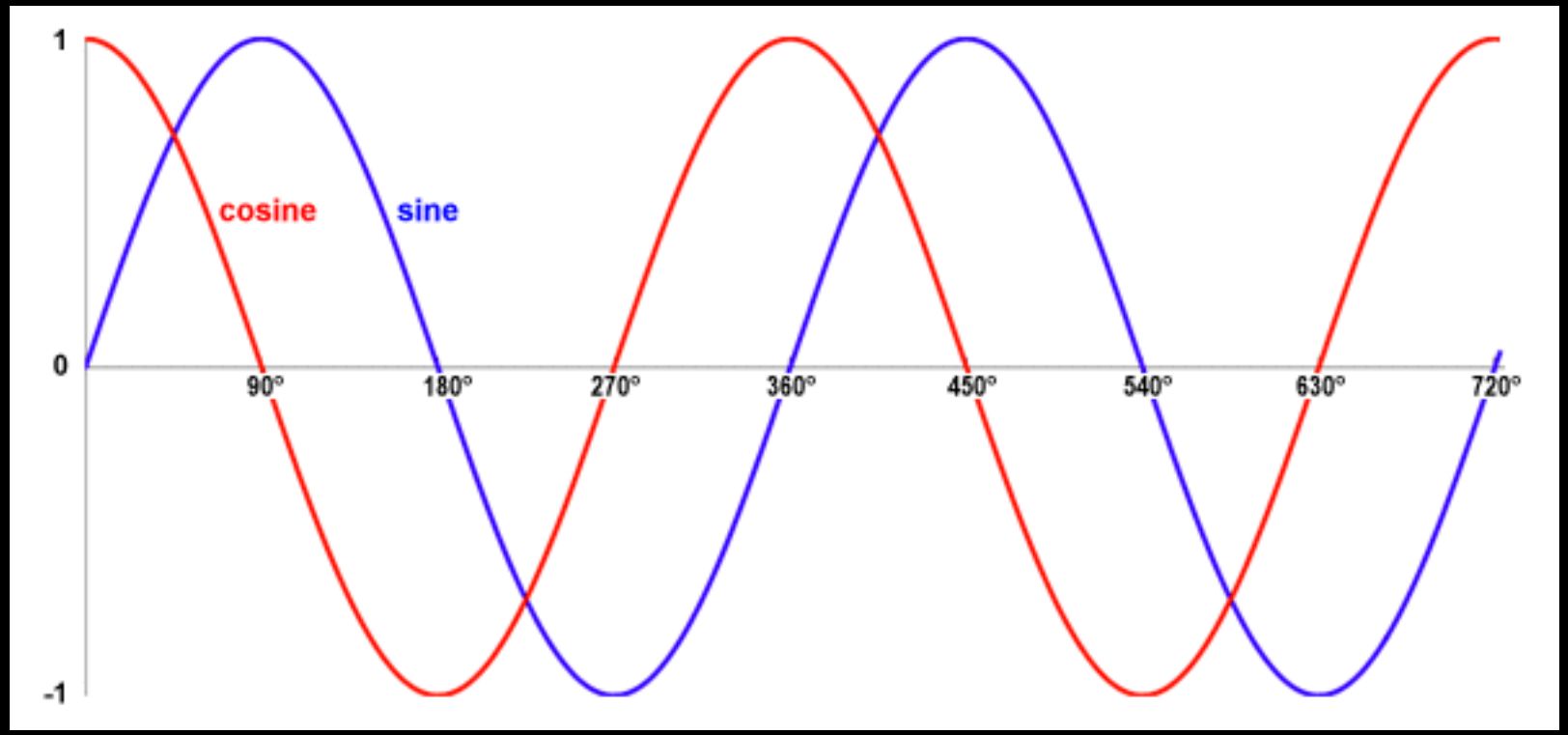
Calculus!

- What is the derivative of $\cos(t)$?



Calculus!

- What is the derivative of $\cos(t)$?
- $-\sin(t)$



More Rules of Calculus!

- Chain rule: $\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$
  - Kind of like fractions cancelling, don't tell mathematicians
- $\frac{df(ax)}{dx} = a\frac{df}{dx}$
- Derivative of derivative, $\frac{d^2f}{dx^2}$
- The derivative of x^n is $\mathrm{n}x^{n-1}$
  - The derivative of a constant is 0
- The derivative of e^x is e^x

How does this relate to motion?

- Velocity is the derivative of position
  - $\frac{dx}{dt} = v(t)$
- Acceleration is the derivative of velocity
  - $\frac{dv}{dt} = a(t)$

Example: Projectile Motion

- If $a(t) = a$, i.e. acceleration is constant, this must mean the velocity is linear in time
  - $v(t) = v_0 + at$
- Likewise, with that velocity function, we can figure out the position by thinking about what this is a derivative of
  - $x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$
  - Note that this is Galileo's result, but with initial position and initial velocity

Newton's Laws in Action

- Elevator accelerating constantly upward
  - $F_G = mg$, F_N
- Effect of air resistance on projectile
  - $F_{air} = -bv$
- Spring system
  - $F_{spring} = -kx$
- Pendulum

Reformulations of Classical Mechanics

- Lagrangian mechanics
  - Why does a ball fall in a parabola?
  - The “right” path
  - $L = T - U$
- Hamiltonian mechanics
  - Transform Lagrangian into something nicer
  - $H = T + U$
  - Why?

$$\frac{\partial L}{\partial f} - \frac{d}{dt}\frac{\partial L}{\partial f'} = 0$$

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} \end{aligned}$$

Next time: Thermodynamics and Optics