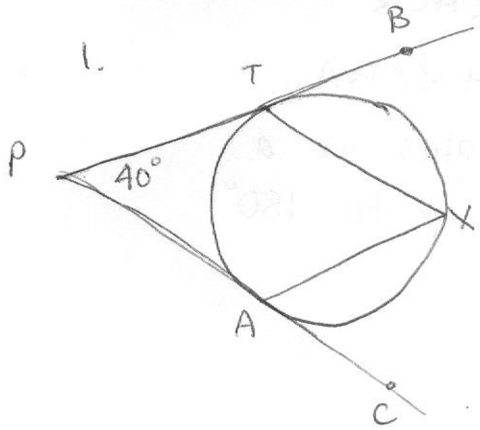
Homework 1 Solutions

1.



Extend lines PT and PA, to points B and C respectively.

Since lines PT and PA are tangent to the circle, $\angle XAC = \frac{\overset{\frown}{XA}}{2}$ and $\angle BTX = \frac{\overset{\frown}{TX}}{2}$. $\angle AXT = \frac{\overset{\frown}{TA}}{2}$, and $\angle TPA = 40°$.

We also know that $\overset{\frown}{TX} + \overset{\frown}{XA} + \overset{\frown}{TA} = 360°$, ①

and $\angle TPA + \angle PAX + \angle AXT + \angle XTP = 360°$. ②

$\angle PAX = 180° - \angle XAC = 180° - \frac{\overset{\frown}{XA}}{2}$

$\angle XTP = 180° - \angle BTX = 180° - \frac{\overset{\frown}{TX}}{2}$

Plug into ②: $\angle TPA + \left(180° - \frac{\overset{\frown}{XA}}{2}\right) + \left(180° - \frac{\overset{\frown}{TX}}{2}\right) + \angle AXT = 360°$

$\angle TPA = 40° \Rightarrow \quad 40° + 360° - \frac{(\overset{\frown}{XA} + \overset{\frown}{TX})}{2} + \angle AXT = 360°$

$\Rightarrow \quad 40° - \frac{(\overset{\frown}{XA} + \overset{\frown}{TX})}{2} + \angle AXT = 0°$

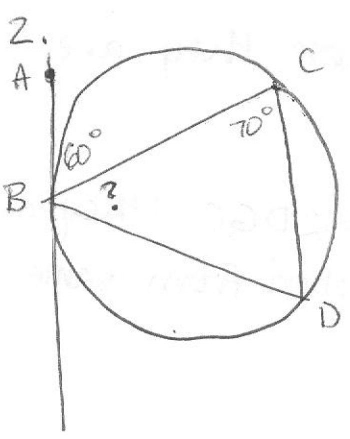
Plug in ①: $\quad 40° - \frac{360° - \overset{\frown}{TA}}{2} + \angle AXT = 0°$

$\Rightarrow \quad 40° - 180° + \frac{\overset{\frown}{TA}}{2} + \angle AXT = 0°$

$\angle AXT = \frac{\overset{\frown}{TA}}{2} \quad \Rightarrow \quad 2(\angle AXT) = 180° - 40° = 140°$

$\Rightarrow \quad \boxed{\angle AXT = 70°}$

2.



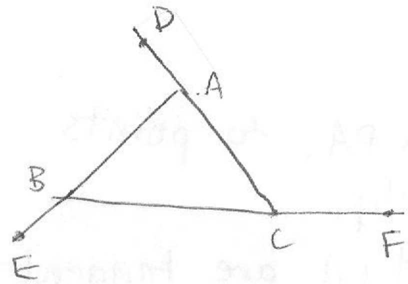
$\angle ABC = 60° \Rightarrow \overset{\frown}{BC} = 2 \cdot 60° = 120°$

$\angle BCD = 70° \Rightarrow \overset{\frown}{BD} = 2 \cdot 70° = 140°$

$\overset{\frown}{CD} = 360° - \overset{\frown}{BC} - \overset{\frown}{BD} = 360° - 120° - 140° = 100°$

$\angle CBD = \frac{\overset{\frown}{CD}}{2} = \frac{100°}{2} = \boxed{50°}$

3.



(a) $\angle FCA = 180^\circ - \angle ACB$

$= \angle BAC + \angle CBA$,

Since the angles in a triangle add up to 180°.

(b) $\angle FCA = 180^\circ - \angle ACB$

$\angle EBC = 180^\circ - \angle ABC$

$\angle DAB = 180^\circ - \angle BAC$

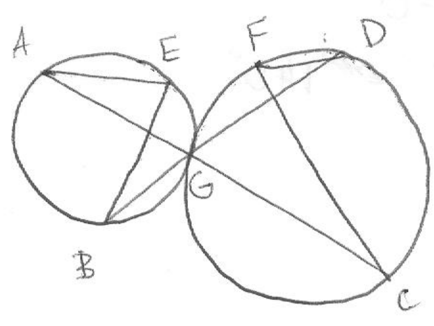
$$\Rightarrow \angle FCA + \angle EBC + \angle DAB = 3 \cdot 180^\circ - (\angle ACB + \angle ABC + \angle BAC)$$

$$= 3 \cdot 180^\circ - 180^\circ$$

$$= 360^\circ$$

Therefore the sum of the exterior angles of a triangle is 360°.

4.



Show: $\angle E = \angle F$.

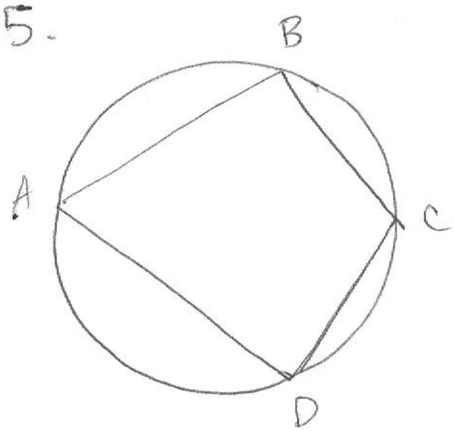
$$\left.\begin{array}{l} \angle E = \dfrac{\overset{\frown}{AB}}{2} \\ \angle AGB = \dfrac{\overset{\frown}{AB}}{2} \end{array}\right\} \Rightarrow \angle E = \angle AGB$$ (they are inscribed from the same arc)

$\angle AGB = \angle DGC$ because they are vertical angles.

$$\left.\begin{array}{l} \angle DGC = \dfrac{\overset{\frown}{DC}}{2} \\ \angle F = \dfrac{\overset{\frown}{DC}}{2} \end{array}\right\} \Rightarrow \angle F = \angle DGC$$ (they are inscribed from same arc)

Thus, $\angle E = \angle AGB = \angle DGC = \angle F$.

5.



$\angle A = 85^\circ = \dfrac{\overset{\frown}{BCD}}{2} \Rightarrow \overset{\frown}{BCD} = 2 \cdot 85^\circ$
$= 170^\circ$

$\overset{\frown}{BAD} = 360^\circ - \overset{\frown}{BCD} = 190^\circ$

$\angle C = \dfrac{\overset{\frown}{BAD}}{2} = \dfrac{190^\circ}{2} = \boxed{95^\circ}$

$\angle B$ and $\angle D$ should also add up to 180° (supplementary).