Statistical Mechanics: Mindblowing Science of Physics for Make Benefit Glorious Learnings of Splash

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What is Statistical Mechanics

- Why you are HERE!
- Physics of everyday life
- Gases, magnets, all sorts of cool stuff
- The “energy" crisis?
- Motivated Quantum Mechanics (will explain more later)
- Win lots of Nobel Prizes (Bose-Einstein Condensates @ MIT)

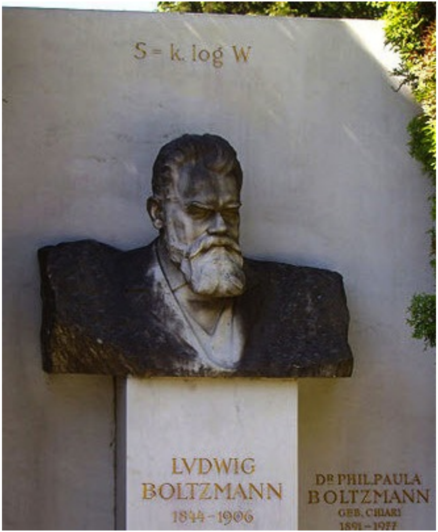
What can you calculate?

- Properties of semiconductors
- Entropic forces (DNA coiling)
- Engines and power plants
- Derive the ideal gas law (will do this)

Stat Mech

- Calculating big things from small things
- Macrostate - The state of the big system (e.g., Pressure, Temperature, . . .)
- Microstate - The microscopic state
- Multiplicity (Ω) - How many microstates give you a certain macrostate?
- Entropy $\rightarrow S = k_B \log \Omega$ (On Boltzmann's tomb)

Founder of Information Theory



Laws of Thermodynamics

- More binding than a subpoena o.O
- 1: You can't win
- 2: Can't even break even
- 3: You have to play
- 0: ???

Stat Mech Explanation

- 1: Conservation of energy
- 2: All microstates are equally probable
- 3: There is a quantum mechanical ground state

Terminology

- Thermal Equilibrium: the state of highest multiplicity
- Extensive: proportional to how much you have (e.g., energy, mass)
- Intensive: independent of how much (e.g., temperature, pressure)
- Note: we want entropy to be extensive

Zeroth Law

- Define temperature
- The property of systems that says the direction in which energy flows
- At the same temperature, no energy flows

Defining Temperature

- Two systems in thermal equilibrium $\rightarrow \Omega = \Omega_1 \Omega_2$
- $S_1(E_1) + S_2(E_2) = S_{tot}(E_1 + E_2)$
- $\frac{\partial S_{tot}}{\partial E_1} = \frac{\partial S_{tot}}{\partial E_2} = 0$
- $$\frac{\partial S_1(E_1)}{\partial E_1} = -\frac{\partial S_2(E_{tot} - E_1)}{\partial E_1} = \frac{\partial S_2(E_2)}{\partial E_2}$$
- Left side is only system 1, right side is only system 2
- Condition for equilibrium
- So,
$$\frac{\partial S(E)}{\partial E} = f(T)$$
- By convention, $f(T) = 1/T$

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What does this mean physically?

- If $\frac{\partial S_1}{\partial E_1} > \frac{\partial S_2}{\partial E_2}$, then total entropy can be increased by shifting energy from System 2 to System 1
- So System 2 is “hotter"
- $f(T)$ should be a decreasing function of T
- We can transfer energy between the systems in two ways: Work or Heating

Work and Heating

- Work: ordered transfer of energy, e.g., a piston moving things to one side
- Heating: unordered transfer of energy
- Note: work doesn't change the total number of microstates
- So

$$\frac{1}{T} = \frac{\partial S}{\partial E}\bigg|_{\text{No work done}}$$

- Negative temperature. Negative is hotter than positive.

Second Law: Applications

- Time moving forward
- Ice cubes melting
- Things not spontaneously cooling down, or speeding up
- How engines work
- ~~Energy~~ Entropy crisis

Donut Break!

The Ideal Gas Law

- $W = p\,dV$. No work done means $dV = 0$
- $dE = T\,dS - p\,dV$
- How do we get the multiplicity...?

Multiplicity

- Why isn't the multiplicity infinite?
- Heisenberg Uncertainty Principle
- $\Delta x \Delta p \geq \left| \langle \psi | \frac{1}{2i} [\hat{x}, \hat{p}] | \psi \rangle \right| = \frac{\hbar}{2} = \frac{h}{4\pi}$
- Phase space is quantized!

Calculating Ω

- Volume of an N-Sphere: $\frac{\pi^{\frac{n}{2}} R^n}{\left(\frac{n}{2}\right)!}$
- (Cause we'll forget that. Look at the board to see us derive the rest!)