

# Exercises for Become a L<sup>A</sup>T<sub>E</sub>Xer!

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If you have trouble figuring out how to typeset one of the following, try to figure it out using the Internet before asking me how to typeset it. Knowing how to find the information you want on the Internet is an important skill. If you're stuck or frustrated, feel free to ask for help.

Note that <http://detexify.kirelabs.org/classify.html> is useful.

## 1 Easy

$$e^{i\pi} + 1 = 0 \tag{1}$$

$$e^{i\theta} = \cos \theta + i \sin \theta \tag{2}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4}$$

$$\vec{L} = \vec{r} \times \vec{p} \tag{5}$$

$$\sqrt[3]{2} \tag{6}$$

$$(x + y)^n = \sum_{r=0}^n \binom{n}{r} x^r y^{n-r} \tag{7}$$

$$\sqrt{\frac{a_1^2 + \dots + a_n^2}{n}} \geq \frac{a_1 + \dots + a_n}{n} \geq \sqrt[n]{a_1 \dots a_n} \geq \frac{n}{\frac{1}{a_1} + \dots + \frac{1}{a_n}} \tag{8}$$

$$|\langle x, y \rangle|^2 \leq \langle x, x \rangle \cdot \langle y, y \rangle \tag{9}$$

$$\text{A1: } \varphi \longrightarrow (\psi \rightarrow \varphi)$$

$$\text{A2: } (\varphi \rightarrow (\psi \rightarrow \theta)) \longrightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \theta))$$

$$\text{A3: } (\neg\varphi \rightarrow \neg\psi) \longrightarrow (\psi \rightarrow \varphi) \tag{10}$$

## 2 Medium

$$1_A = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \quad (11)$$

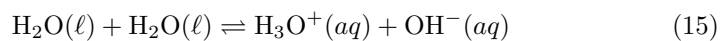
$$\underbrace{n \uparrow \cdots \uparrow}_n n = n \rightarrow n \rightarrow n \quad (12)$$

$$1 \uparrow 1 = {}^1 1 = 1$$

$$2 \uparrow 2 = {}^2 2 = 4$$

$$3 \uparrow \uparrow 3 = {}^3 3 = 3 \uparrow 3 \uparrow 3 = \underbrace{3^{3^{3^{3^{3^{\cdots}}}}}}_{3^{3^3} \text{ threes}} \quad (13)$$

$$\frac{d}{dx} f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (14)$$



$$\Gamma(n+1) \stackrel{\text{def}}{=} \int_0^\infty e^{-t} t^n dt \quad (16)$$

$$\gcd(n, m \bmod n); \quad x \equiv y \pmod{b}; \quad x \equiv y \pmod{c}; \quad x \equiv y \pmod{d} \quad (17)$$

In the following, note the bold symbols.

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{aligned} \quad (18)$$

For the following exercise, you will need to `\usepackage{esint}` to get the symbol  $\oiint$ .

$$\begin{aligned}
 \oiint_{\partial V} \mathbf{E} \cdot d\mathbf{A} &= \frac{Q(V)}{\epsilon_0} \\
 \oiint_{\partial V} \mathbf{B} \cdot d\mathbf{A} &= 0 \\
 \oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} &= -\frac{\partial \Phi_{B,S}}{\partial t} \\
 \oint_{\partial S} \mathbf{B} \cdot d\mathbf{l} &= \mu_0 I_S + \mu_0 \epsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}
 \end{aligned}
 \tag{19}$$

You might find the environments `bmatrix` and `pmatrix` useful for the following exercises.

$$\rho_\theta = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}
 \tag{20}$$

$$\left[ \begin{array}{c|ccc} 1 & 0 & \dots & 0 \\ \hline 0 & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & * & \dots & * \end{array} \right] = \left[ \begin{array}{c|ccc} 1 & 0 & \dots & 0 \\ \hline 0 & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & * & \dots & * \end{array} \right]
 \tag{21}$$

Note the locations of the bounds on the summation in the following exercise.

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N p_i (x_i - \bar{x})^2} = \sqrt{\frac{\sum_{i=1}^N p_i (x_i - \bar{x})^2}{N}}
 \tag{22}$$

$$\varphi(n) = n \cdot \prod_{\substack{p|n \\ p \text{ prime}}} \left(1 - \frac{1}{p}\right)
 \tag{23}$$

If you `\usepackage{mathtools}`, you can make it look like

$$\varphi(n) = n \cdot \prod_{\substack{p|n \\ p \text{ prime}}} \left(1 - \frac{1}{p}\right)
 \tag{24}$$

Parts of the following exercise require you to `\usepackage{mathtools}`.

$${}_{12}C_2^{5+} \quad {}_2C_2^{5+} \quad {}_{12}C_2^{5+} \quad {}_{14}C_2^{5+} \quad {}_2C_2^{5+}
 \tag{25}$$

In the following, note the size of the /, and the spacing on the sides of the |.

$$\mathbb{Q} \cong \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\} / \sim$$

$$\frac{a}{b} \sim \frac{c}{d} \iff ad - bc = 0 \tag{26}$$

Notice both the horizontal and vertical spacing in the following exercise.

$$\begin{aligned} 1 \uparrow 1 &= {}^1 1 = 1 \\ 2 \uparrow\uparrow 2 &= {}^2 2 = 4 \\ 3 \uparrow\uparrow\uparrow 3 &= {}^3 3 = 3 \uparrow\uparrow 3 \uparrow\uparrow 3 = \underbrace{3^{3^{3^{3^{3^{\dots^3}}}}}}_{3^{3^3} \text{ threes}} \end{aligned} \tag{27}$$

### 3 Hard

The command `\newcommand{\name}[n][default]{definition}` defines a new command, where  $n$  is the number of parameters and  $default$  is the default value for the first parameter. Parameters in braces ( $\{ \}$ ) are required, and parameters in square brackets ( $[ ]$ ) are optional. The parameters can be referred to via  $\#1$ ,  $\#2$ ,  $\dots$ ,  $\#9$ . Using `\newcommand{\mathset}[2][default value for the first argument]{command definition}`, define a command `\mathset` that acts as follows. Notice, in particular, the sizes of the middle bar, the sizes of the braces, and the spacing between the middle bar and the things on each side.

<code>\mathset{1}</code>	gives $\{1\}$
<code>\mathset[x]{0\leq x\leq 1}</code>	gives $\{x \mid 0 \leq x \leq 1\}$
<code>\mathset[(x)_i]{\sum_i x_i \in A}</code>	gives $\left\{ (x)_i \mid \sum_i x_i \in A \right\}$
<code>\mathset[\sum_{i=1}^{\infty} n^{-s}]{n \in A}</code>	gives $\left\{ \sum_{i=1}^{\infty} n^{-s} \mid n \in A \right\}$
<code>\mathset{}</code>	gives $\{ \}$
<code>\mathset[\frac{1}{1+\frac{1}{x}}]{x \in A}</code>	gives $\left\{ \frac{1}{1+\frac{1}{x}} \mid x \in A \right\}$
<code>\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{x}}}]{x \in A}</code>	gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{x}}} \mid x \in A \right\}$
<code>\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}]{x \in A}</code>	gives $\left\{ 1 + \frac{1}{1+\frac{1}{1+\frac{1}{x}}} \mid x \in A \right\}$
<code>\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}]{x \in A}</code>	gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}} \mid x \in A \right\}$
<code>\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}}]{x \in A}</code>	gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}} \mid x \in A \right\}$
<code>\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}}}]{x \in A}</code>	gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}}} \mid x \in A \right\}$

## 4 Insane

Write a command `\outputcode` which outputs the code of the document being typeset.

The following exercise idea is taken from the `TEXbook`. Define a command `\primes{n}` which typesets the first  $n$  primes. For example, `\primes{30}` should give 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, and 113.

## 5 Diabolical

The Ackermann function is defined as

$$A(m, n) = \begin{cases} n + 1 & \text{if } m = 0 \\ A(m - 1, 1) & \text{if } m > 0 \text{ and } n = 0 \\ A(m - 1, A(m, n - 1)) & \text{if } m > 0 \text{ and } n > 0 \end{cases}$$

Define a command, `\ackermann{m}{n}`, which computes the Ackermann function and displays the intermediate values. For an extra challenge, ensure that it only displays each sub-value only once (per command call; a second `\ackermann{m}{n}` should typeset the same thing as the first one).

For example, `\ackermann{2}{2}` should display

$$\begin{aligned} A(2, 2) &= A(1, A(2, 1)) \\ &= A(1, A(1, A(2, 0))) \\ &= A(1, A(1, A(1, 1))) \\ &= A(1, A(1, A(0, A(1, 0)))) \\ &= A(1, A(1, A(0, A(0, 1)))) \\ &= A(1, A(1, A(0, 2))) \\ &= A(1, A(1, 3)) \\ &= A(1, A(0, A(1, 2))) \\ &= A(1, A(0, A(0, A(1, 1)))) \\ &= A(1, A(0, A(0, 3))) \\ &= A(1, A(0, 4)) \\ &= A(1, 5) \\ &= A(0, A(1, 4)) \\ &= A(0, A(0, A(1, 3))) \\ &= A(0, A(0, 5)) \\ &= A(0, 6) \\ &= 7 \end{aligned}$$