e and the Complex Numbers: Syllabus

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HSSP Spring 2008

1 Course Description

$$e^{i\pi} + 1 = 0$$

So said Euler in 1748. This equation, considered by many as the most amazing equation ever, is known as "Euler's Identity".

Doesn't it disturb you a bit? Like, what does it even mean to raise something to an imaginary power? And what's so special about e anyway? It's not like 2.718 measures some elementary geometric ratio the way π does, though the appearance of π in there is spooky too.

We'll take a little trip into the wonderful world of complex analysis in an attempt to reach an understanding of this equation.

2 Prerequisites

Familiarity with high school algebra (you should be able to use exponents, polynomials, and the sine and cosine functions) and some idea of what derivatives and integrals are.

3 Approximate Schedule of Topics

Day 1 — Spark!

- Ridiculous observations about e
- Exponents and logarithms

Day 2 — The complex numbers at a glance

- Trigonometry in the complex plane
- de Moivre's formula

- The cis function

Day 3 — Calculus revisited

- Review of derivatives and integrals in the real numbers
- Observations about the derivative of e^x
- Deriving the limit definition of e

Day 4 — Taylor series

- Convergence and Taylor series in the real numbers
- Taylor expansions of the exponential and trigonometric functions

Day 5 — Dabbling in multivariable calculus

- Partial derivatives
- Integrating things along curves
- Derivatives of real-to-complex functions

Day 6 — Complex analysis

- Limits and derivatives in the complex numbers
- The Cauchy-Riemann equations
- Observations about complex-differentiable functions

Day 7 — More complex analysis

- Integrating complex-differentiable functions around loops
- Convergence and Taylor series in the complex numbers

Day 8 — Back to e

- Extending the exponential function to the complex numbers
- Proofs of Euler's Identity

If extra time remains — Bonus material

- Logarithms and trigonometric functions on the complex numbers
- The continued fraction expansion of e