11483: Introduction to Modern Physics Lecture-1

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1 Introduction

"Modern physics is the post-Newtonian conception of physics. It implies that classical descriptions of phenomena are lacking, and that an accurate description of nature requires theories to incorporate elements of quantum mechanics or Einsteinian relativity, or both. In general, the term is used to refer to any branch of physics either developed in the early 20th century and onwards, or branches greatly influenced by early 20th century physics."

1.1 Timeline of Modern Physics

1900 - Formula for Black Body radiation: Max Planck
1905 - Special relativity: Albert Einstein
1905 - Photoelectric effect: Albert Einstein
1911 - Equivalence Principle: Albert Einstein
1911 - Discovery of the Atomic Nucleus: Ernest Rutherford
1913 - Bohr Model of the atom: Niels Bohr
1916 - General Relativity: Albert Einstein
1923 - Matter waves: Louis de Broglie
1925 - Matrix Mechanics: Werner Heisenberg
1926 - Schrodinger Equation: Erwin Schrodinger
1927 - Uncertainty Principle: Werner Heisenberg
1928 - Dirac Equation: Paul Dirac
1948 - Theory of Quantum Electrodynamics: Richard Feynman

2 Special Relativity : Introduction

2.1 Newtonian Mechanics

Newton's first law : In an inertial reference frame, an object at rest remains at rest and an object moving with a constant velocity continues to move with that velocity until acted upon by a force.

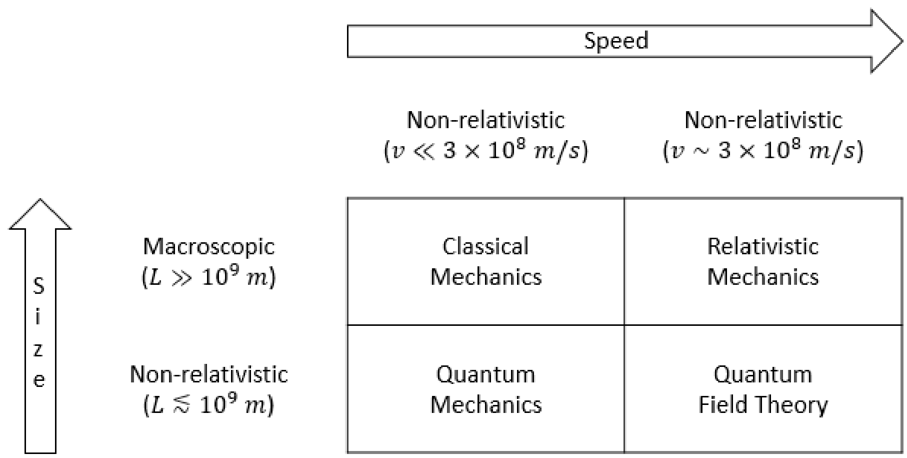

Figure 1: Realms of Physics

Newton's second law : In an inertial reference frame, the net external force on an object is equal to rate of change of momentum.

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \tag{1}$$

If the mass of the object does not change, the net external force equals the mass of that object multiplied by the acceleration of the object.

$$\mathbf{F} = m\mathbf{a} \tag{2}$$

Naively, the first law appears to be a corollary of the second law because no net force implies no acceleration which in turn implies that the velocity of the object does not change.

However, the first law is actually a statement defining an inertial reference frame and the second law holds only in such a frame.

2.2 Galilean Transformation

Consider an inertial reference frame F and another inertial reference frame F' moving at a velocity $u\hat{x}$ with respect to F. The origins of these two frames are coincident at $t = t' = 0$.

The co-ordinate transformation between these two frames is given by the Galilean transformations,

$$\begin{aligned} t' &= t \\ x' &= x - ut \\ y' &= y \\ z' &= z \end{aligned} \tag{3}$$

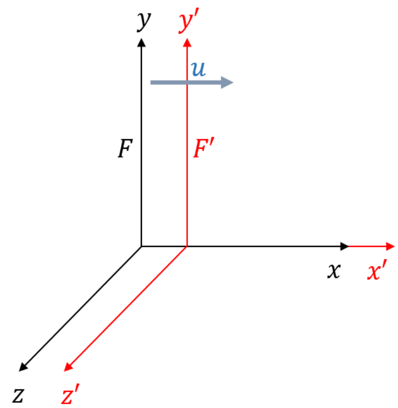

Figure 2: Sketch of the inertial reference frames

and analogously,

$$\begin{aligned} t &= t' \\ x &= x' + ut' \\ y &= y' \\ z &= z' \end{aligned} \tag{4}$$

Hence, the law of velocity addition is

$$\mathbf{v}' = \mathbf{v} - u\hat{x} \tag{5}$$

2.3 Postulates of Special Relativity

1. The Principle of Relativity : The laws of physics are the same in all inertial frames of reference.
2. The Constancy of Speed of Light in Vacuum : The speed of light in vacuum has the same value c in all inertial frames of reference.

2.4 Deriving the Lorentz Transformations

Let us consider the simplified case with only one spatial direction, x.

The transformations must be linear in both t and x. This is because the time intervals and object lengths are independent of the origin so t' and x' cannot depend on any non-linear function of t and x.

$$\begin{aligned} t' &= At + Bx \\ x' &= Mx + Nt \end{aligned} \tag{6}$$

Thus, a velocity in F' is related to a velocity in F via

$$v' = \frac{\Delta x'}{\Delta t'} = \frac{M\Delta x + N\Delta t}{A\Delta t + B\Delta x} = \frac{Mv + N}{A + Bv} \tag{7}$$

Since F' moves with velocity u with respect to F, $v = 0$ corresponds to $v' = -u$

$$-u = \frac{N}{A} \tag{8}$$

$$N = -Au \tag{9}$$

Also, $v = u$ corresponds to $v' = 0$

$$0 = Mu + N \tag{10}$$

$$M = -\frac{N}{u} = A \tag{11}$$

Hence,

$$\begin{aligned} t' &= At + Bx \\ x' &= A(x - ut) \end{aligned} \tag{12}$$

Since speed of light in vacuum is constant, $v = c$ corresponds to $v' = c$

$$c = \frac{A(c - u)}{A + Bc} \tag{13}$$

$$B = -\frac{Au}{c^2} \tag{14}$$

Also, $v = -c$ corresponds to $v' = -c$

$$-c = \frac{-A(c + u)}{A - Bc} \tag{15}$$

$$B = -\frac{Au}{c^2} \tag{16}$$

Hence,

$$\begin{aligned} t' &= A(t - ux/c^2) \\ x' &= A(x - ut) \end{aligned} \tag{17}$$

In accordance with principle of relativity, the inverse transformation i.e. transformation from x' and t' to x and t must be generated by replacing u by $-u$. This is because F moves at velocity $-u$ with respect to F'.

$$\begin{aligned} t &= A(t' + ux'/c^2) \\ x &= A(x' + ut') \end{aligned} \tag{18}$$

$$t = A^2[(t - ux/c^2) + u(x - ut)/c^2] = A^2 t(1 - u^2/c^2) \tag{19}$$

$$A = 1/\sqrt{1 - u^2/c^2} \equiv \gamma(u) \tag{20}$$

Thus, we have the Lorentz transformations

$$\begin{aligned} t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} \\ x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} \end{aligned} \tag{21}$$

and analogously,

$$\begin{aligned} t &= \frac{t' + ux'/c^2}{\sqrt{1 - u^2/c^2}} \\ x &= \frac{x' + ut'}{\sqrt{1 - u^2/c^2}} \end{aligned} \tag{22}$$

In the limiting case $u \ll c$, using binomial approximation

$$\gamma(u) \approx 1 - u^2/2c^2 \approx 1 \tag{23}$$

$$\begin{aligned} t' &\approx t \\ x' &\approx x - ut \end{aligned} \tag{24}$$

as expected.

The relativistic law of velocity addition is

$$v' = \frac{v - u}{1 - uv/c^2} \tag{25}$$

$$v = \frac{v' + u}{1 + uv'/c^2} \tag{26}$$

A Functions

A function is rule that generates outputs based on the inputs given to it.

For instance, consider the function $f(x) = x^2$. Here, x is the input and $f(x)$ is the output. For the input $x = 1$, we get the output $f(1) = 1$. Similarly, $x = 2$ gives $f(2) = 4$ and so on.

The input of a function is called the argument. A function can have a single argument, like $f(x) = x^2$, or multiple arguments, like $f(x_1, x_2) = x_1 + x_2$.

A linear function in one argument is of the form $f(x) = ax + b$ where a and b are arbitrary. The graph of such a function is a straight line, so the function is called linear. More generally, a linear function in n argument is of the form $f(x_1, x_2, ...x_n) = a_1x_1 + a_2x_2 + ...a_nx_n + b$ where a_1, a_2, $...a_n$ and b are arbitrary.

B Binomial Approximation

For any real number r and any x such that $|x| \ll 1$, we have the binomial approximation

$$(1 + x)^r \approx 1 + rx \tag{27}$$

This can be seen as a generalization of the binomial theorem, which states that for any natural number n and arbitrary real numbers a and b

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + ...b^n \tag{28}$$