

# Flocking behavior

Ekaterina Kryuchkova

July 18, 2020

## 1 Cucker-Smale Model

Here is a simple framework for modeling self-organizing flocking behavior of birds. Here I summarize the description from Motsch [1]. Simple flocking behavior can be described by the position and the velocity of each bird.

$$\frac{d\vec{x}_i}{dt} = \vec{v}_i, \quad (1)$$

and

$$\frac{d\vec{v}_i}{dt} = \frac{\alpha}{N} \sum_{j=1}^N \phi_{ij}(\vec{v}_j - \vec{v}_i), \quad (2)$$

where there are  $N$  birds total, and  $\Sigma$  denotes a sum. The function  $\phi_{ij}$  controls the strength of the birds' influence on one another. One choice for this function is

$$\phi_{ij} = \phi(|\vec{x}_j - \vec{x}_i|), \quad (3)$$

where  $\phi(\cdot)$  is a strictly positive decreasing function.

### 1.1 Questions to think about.

1. What does the term  $(\vec{v}_j - \vec{v}_i)$  do? (Hint: when is it zero? nonzero?)
2. What effect does the function  $\phi(|\vec{x}_j - \vec{x}_i|)$  have?
3. Describe how the term  $\phi_{ij}(\vec{v}_j - \vec{v}_i)$  influences the velocity.
4. Why do we divide by  $N$ ?
5. What effect does the parameter  $\alpha$  have?

## References

- [1] Motsch, S. and Tadmor, E., 2011. A new model for self-organized dynamics and its flocking behavior. Journal of Statistical Physics, 144(5), p.923.