Intro to Blindfold Cubing

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1 Background

Blindfold cubing looks really hard. It's not. You only need to learn a few concepts and a few algorithms to be able to solve a cube blindfolded. I believe any of you can learn to solve a cube blindfolded, although I will admit that it will take patience and practice.

The method we're going to learn is called 3OP, which stands for 3-cycle Orientation Permutation. Up until a few years ago, 3OP was the preferred method used by the world's best blindfold cubers, yet is still a simple and straightforward method.

2 The Gameplan

So, what's going to go on in our blindfold solve? Any blindfold solve is broken up into two parts: memorization and resolution. During the memorization phase, you do just that - memorize the cube. Once you're done memorizing, you put on a blindfold, place the cube behind your back, close your eyes, whatever. Then, you (hopefully) solve the cube without looking at it. If you plan on timing yourself, your time is the total time it takes to memorize and solve the cube.

In 3OP, there are four steps: corner orientation (CO), edge orientation (EO), corner permutation (CP), and edge permutation (EP). Each cubie has an orientation and a permutation. Regarding orientation, note that an edge piece can be flipped in one of two ways, and a corner piece can be rotated in one of three ways. Permutation refers to the location of a piece; each piece must be permuted, or moved, to its proper location. Each of these steps is independent of the others, meaning, for example, that fixing the edge orientation will not affect the corner permutation. (There is a small exception to this if there is permutation parity.)

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3 Setup

Pick two adjacent colors of the cube that you like. One of these colors will be your top face, and the other will be your front face. **For every blindfold solve, you will start by putting the cube in this orientation.**

Now, we assign a number to each cubie:

#	Corners	Edges
1	UFL	UF
2	UBL	UL
3	UBR	UB
4	UFR	UR
5	DFL	FL
6	DBL	BL
7	DBR	BR
8	DFR	FR
9		DF
10		DL
11		DB
12		DR

This is the numbering scheme we'll use for this guide. You don't have to use this setup; you can use any numbers, letters, words, images, etc. that you prefer. However, it's important that you stay consistent once you've decided on how you want to label your pieces.

Once you've decided on the scheme you like, test yourself! Scramble your cube, then see if you can associate the proper label and location with each piece.

4 Conjugation

A **conjugate** is an algorithm in the form XYX^{-1}. Here, Y refers to some algorithm we want to use. X refers to some **set-up moves** and X^{-1} refers to the reverse of these same set-up moves.

Example. Let's say that we want to move three edge pieces which are located at positions 2, 4, and 10, and we know an algorithm that will move the pieces at positions 2, 3, and 4. In this example, Y will be this algorithm. However, we need to set up our three edge pieces. We can move the edge piece at location 10 to location to by doing the set-up moves $\mathrm{D'B^2}$. We can then perform Y, which will affect only the pieces in locations 2, 3, and 4 (which were originally the pieces in locations 2, 4, and 10). Afterwards, we then perform $X^{-1} = \mathrm{B^2D}$ to finish. The net result of all of these moves is that we have moved only the pieces at 2, 4, and 10. □

Conjugation is a very important idea in blindfold cubing, but don't worry if this explanation doesn't make sense right now. It will be much clearer when you try it in context.

5 Orientation

5.1 Edge Orientation

5.1.1 Definitions

As mentioned before, each edge piece has two orientations. One is correct, one isn't. However, we can have different definitions for what it means to be a "correct" orientation. For this guide, we will say that an edge is correctly oriented if it can be reached from a solved cube within the $(\mathrm{UDFBR}^2\mathrm{L}^2)$ group. More simply, this means an edge piece is correctly oriented if we can return it to its original position without using any quarter turns on the R and L faces.

There's an easy method to check whether or not an edge is correctly oriented:

1. If the edge piece is in the U or D layer,
   - If the edge piece has a U or D sticker, it is correctly oriented if this sticker is on the U or D face. It is incorrectly oriented if this sticker is on the F, B, R, or L face.
   - Otherwise, the piece must have an F or B sticker. It is correctly oriented if this sticker is on the F, B, R, or L face. It is incorrectly oriented if this sticker is on the U or D face.
2. If the edge piece is in the middle layer,
   - If the piece has a U or D sticker, it is correctly oriented if this sticker is on the R or L face. It is incorrectly oriented if this sticker is on the F or B face.
   - Otherwise, the piece must have an F or B sticker. It is correctly oriented if this sticker is on the F or B face. It is incorrectly oriented if this sticker is on the R or L face.

Incorrectly oriented edges come in pairs, so you should always have an even number of incorrectly oriented edges.

5.1.2 Solving

Finally, something that isn't a definition or theory! To complete the edge orientation step, you will first **memorize which edges are incorrectly oriented**. We then use this algorithm, which flips the edges in locations 1 and 3:

$$\mathrm{M'UM'UM'U^2MUMUMU^2}.$$

We then **orient pairs of edges** using the algorithm above. What if the edges we want to flip aren't in locations 1 and 3? This is where conjugation comes in.

Example. Suppose we want to flip edges 5 and 11. We want to use a conjugate; an algorithm in the form XYX^{-1}. We know that we will want to use $\mathrm{Y} = \mathrm{M'UM'UM'U^2MUMUMU^2}$.

First, we set-up our edges into the right places; if we use $\mathrm{X} = \mathrm{FB}^2$, the edges that were previously at locations 5 and 11 are now at locations 1 and 3. Next, we execute Y. Finally, we do $\mathrm{X}^{-1} = \mathrm{B^2F'}$ to put everything back to where it originally was. □

So, to be more specific, we **orient pairs of edges, using conjugation if needed**.

There are many other algorithms that will flip multiple edges, which can be found online. They aren't necessary; it is possible to orient all edges using only the algorithm above. However, these algorithms will speed up your solves if you learn them. As an example, this algorithm flips all edges on the top layer:

$$(\mathrm{M'U})^4(\mathrm{MU})^4 = \mathrm{M'UM'UM'UM'UMUMUMUMU}.$$

5.2 Corner Orientation

5.2.1 Definitions

Similar to edges, corners have orientations as well. However, each corner has three orientations rather than two. Each corner will have either be correctly oriented, rotated clockwise, or rotated counterclockwise. For corners, a "correct" orientation means that the U or D sticker is on the U or D face.

Incorrectly oriented corners come in pairs or triples. More specifically, they are either

1. A pair of oppositely oriented corners. One corner needs to be rotated clockwise, the other needs to be rotated counterclockwise.
2. A triple of similarly oriented corners. These three corners either all need to be rotated clockwise, or all need to be rotated counterclockwise.

5.2.2 Solving

We have two algorithms this time; one for clockwise rotation and one one for counterclockwise rotation. (You can actually get away with knowing just one of these two algorithms. If you rotate a corner clockwise twice, it's the same as rotating it counterclockwise once.)

$$\begin{array}{rl} \text{clockwise} & (\mathrm{D'R'DR})^2 = \mathrm{D'R'DRD'R'DR} \\ \text{counterclockwise} & (\mathrm{R'D'RD})^2 = \mathrm{R'D'RDR'D'RD} \end{array}$$

Both of these algorithms will rotate the corner at location 4 in the desired direction. However, **they will also mess up other pieces**. This is why we must orient corners in pairs or triples, as mentioned before.

So, we start by **memorizing which corners are incorrectly oriented**. We also need to make sure that we **memorize which direction the incorrect corners need to be rotated**.

To actually orient the corners, we have two parts: orient any pairs of corners, then orient any triples of corners.

We'll start with a pair of corners. As a reminder, this means that one corner needs to be rotated clockwise, and the other needs to be rotated counterclockwise. Again, we use conjugation. X will be any sequence of set-up moves that **place both corners on the U face**. Then, do any number of U turns until one of these corners is at position 4. **Remember how many U turns you just did!** Then, use the appropriate algorithm above to **orient that corner**. Again, do any number of U turns until the other corner is at position 4. **Remember the total number of U turns you have done!** As before, use the appropriate algorithm above to **orient the second corner**. Now, we want to **return the corners to their original positions**. To do this, turn the U face until the total number of U turns you have done is a multiple of 4. Then, do X^{-1}.

Example. We want to rotate the corner at location 1 clockwise, and the corner at location 7 counterclockwise. We bring both corners to the U face by using $\mathrm{X} = \mathrm{R}^2$. We now have the corner originally at location 7 at location 4. To rotate it counterclockwise, we do $(\mathrm{R'D'RD})^2 = \mathrm{R'D'RDR'D'RD}$. Then, we want to move the other corner to location 4. We can do this by doing U^{-1} or U^3, they are the same thing. We have done -1 or 3 U turns, depending on how you want to look at it. We then rotate that corner clockwise with $(\mathrm{D'R'DR})^2 = \mathrm{D'R'DRD'R'DR}$. Now, we need to put the corners back where they were. Doing 1 U turn makes our total number of U turns divisible by 4. We then do $\mathrm{X}^{-1} = \mathrm{R}^2$ to finish. □

Now, for a triple of corners that all need to be rotated the same direction. We start with X, a sequence of set-up moves that **place all three corners on the U face**. Do any number of U turns until one of these corners is at position 4; **remember how many U turns you've done**. Then, **orient that corner**. **Repeat for the other two corners.** As before, we now **return the corners to their original positions**. We turn the U face until the total number of U turns you have done is a multiple of 4. Then, do X^{-1}.

Example. We want to rotate the corners at locations 2, 5, and 6 all counterclockwise. We bring all corners to the U face by doing $\mathrm{X} = \mathrm{UL}^2$. We do **not** count this U turn, since it is part of X. The corners we want to orient are now at locations 1, 2, and 3. We do a U turn (1 total U turn) to bring the corners to locations 2, 3, and 4. Then, we orient a corner with $(\mathrm{R'D'RD})^2 = \mathrm{R'D'RDR'D'RD}$. We then do another U turn (2 total U turns) to bring the corners to locations 3, 4, and 1, and orient again with $(\mathrm{R'D'RD})^2 = \mathrm{R'D'RDR'D'RD}$. We have one last corner to orient. Another U turn (3 total) brings the corners to 4, 1, and 2; again, we orient with $(\mathrm{R'D'RD})^2 = \mathrm{R'D'RDR'D'RD}$. To return the corners to their original positions, we do one U turn (to bring the total to 4). We then finish with $\mathrm{X}^{-1} = \mathrm{L}^2\mathrm{U'}$. □

Just as with edges, there are other algorithms that will orient different sets of corners that you can look up if interested.

6 Permutation

6.1 Cycles

For permutation, we will use cycles. Rather than talk about more theory, let's jump into what you need to know.

We need to be able to determine the cycles in a scrambled cube. To do this, we use the **cycle decomposition algorithm**:

1. Find the smallest number that has not been written (the first time this number is 1).
   (a) If this number exists, write "(" and then that number.
   (b) If all numbers have been written, stop.
2. Find the last number that was written. Determine to which spot this piece needs to be moved.
   (a) If the number of this spot has not been written, write it down and repeat step 2.
   (b) If the number of this spot has been written, write ")" to end the cycle. Go to step 1.

Example. In this example, we will only write out the cycles for the corners. In practice, you will need to do this for the corners and the edges. We'll use this scramble:

$$\mathrm{R^2F^2D'L^2B^2U'R^2B^2F^2D^2L^2D'B^2U'R'FR'L'UBDR'FDU'}.$$

We start with the corner in location 1. This corner should go to location 4. The corner there should go to location 6, which should go to location 8, which should go to location 1. We've reached a previously written number, so this is the end of the first cycle. Writing out the entire cycle, we have (1468).

Next, we start with the first unwritten corner, the one in location 2. This corner goes to location 5, which goes to location 7, which goes back to location 2. Our second cycle is thus (257).

Our last cycle ends up simply being (3).

Putting these all together, we have (1468)(257)(3). □

For permutations, we will memorize both edges and corners as cycles. Note that if we have a cycle with just one piece in it, such as (3) in the example, that piece is already in its proper place. As such, we don't have to remember it.

To actually permute the pieces, we use 3-cycles. These are algorithms that move 3 pieces at a time. It's important to know the **cycle reduction rule**: a cycle of length 3 or longer, when its first 3 pieces are cycled, loses the second and the third number.

Example. We'll use the cycle from before, (1468)(257). (We discard the 3 since it's a single number.) We will cycle the first three pieces in a cycle: 1, 4, and 6. After doing so, the cycle reduction rule tells us that our new cycle will become (18)(257). (18) only has two numbers, so we can't use 3-cycles anymore. We use another 3-cycle on 2, 5, and 7, giving us (18)(2), which is the same as (18).

As such, we've solved all but 2 corners. This example actually has permutation parity, which we will need to fix to solve the last two corners. This will be covered later. □

That's the theory behind cycles. Let's get back to solving the cube.

6.2 Corner Permutation

We start with two algorithms (although you can get away with just one of them).

$$\begin{aligned}(143) &= \mathrm{RB'RF^2R'BRF^2R^2} \\ (124) &= \mathrm{L'BL'F^2LB'L'F^2L^2}\end{aligned}$$

(143) moves the corner at location 1 to location 4, the corner at location 4 to location 3, and the corner at location 3 to location 1. (124) similarly cycles the corners at locations 1, 2, and 4.

As before, we will use conjugation. However, we have a restriction now! We've already oriented all the corners, and we don't want to accidentally unorient them. Formally, our set-up moves will be restricted to the $(\mathrm{UDF^2B^2R^2L^2})$ group. This simply means that we cannot use quarter F, B, R, or L turns in our set-up moves. If we turn any of these faces, we must do a 180° rotation.

Example. We want to solve the cycle (178). We start by **setting up the corners** on the U face. We can do this with the set-up move(s) $\mathrm{X} = \mathrm{R}^2$. The corner previously at location 7 is now at location 4, and the corner previously at location 8 is now at location 3. So, the cycle we want to do is (143), which we have an algorithm for! So, we **permute the corners**, in this case using the algorithm $\mathrm{RB'RF^2R'BRF^2R^2}$. To finish, we **return the corners to their original positions** by doing $\mathrm{X}^{-1} = \mathrm{R}^2$. □

If we have a cycle of length longer than 3, we may have to use multiple 3-cycles, remembering to use the cycle reduction rule as necessary.

Here are some additional algorithms that can make some 3-cycles easier.

$$\begin{aligned}(173) &= \mathrm{(R^2DR^2D'R^2U^2)^2 = R^2DR^2D'R^2U^2R^2DR^2D'R^2U^2} \\ (375) &= \mathrm{(R^2U'R^2UR^2D^2)^2 = R^2U'R^2UR^2D^2R^2U'R^2UR^2D^2}\end{aligned}$$

After we finish all the 3-cycles we can, we will sometimes have multiple 2-cycles left over. That is, we might have something like (13)(24)(56), which we cannot reduce using 3-cycles. We can only reduce these 2-cycles in pairs. Here are two algorithms for doing so.

$$
\begin{aligned}
(13)(24) &= \mathrm{M^2UM^2U^2M^2UM^2U^2} \\
(14)(23) &= \mathrm{xUR'U'LURU'r^2U'RULU'R'Ux}^1
\end{aligned}
$$

Example. We want to solve (15)(28). We **set up the corners** with $\mathrm{X} = \mathrm{DR}^2$. The corner previously at 5 is now at 3, and the corner previously at 8 is now at 4. So, we then **permute the corners** with the algorithm $\mathrm{M^2UM^2U^2M^2UM^2U^2}$. Finally, we **return the corners** with $\mathrm{X}^{-1} = \mathrm{R^2D'}$. □

6.3 Edge Permutation

Edge permutation is exactly the same as corner permutation. Again, we'll start with some algorithms:

$$
\begin{aligned}
(243) &= \mathrm{R^2U'R'U'RURURU'R} \\
(234) &= \mathrm{R'UR'U'R'U'R'URUR^2}
\end{aligned}
$$

Our restriction for conjugation is different for edges than for corners. Previously, we defined correct edge orientation to be one that can be reached with $(\mathrm{UDFBR^2L^2})$. This provides the restriction for our set-up moves. We can turn any face, but any R or L turns must be 180° turns.

Example. We want to solve the cycle (167). We start by **setting up the edges** on the U face. We can do this with the set-up move(s) $\mathrm{X} = \mathrm{B'UB^2}$. The corner previously at location 1 is now at location 2, the corner previously at location 6 is now at location 4, and the corner previously at location 7 is now at location 3. So, the cycle we want to do is (243), which we have an algorithm for! So, we **permute the edges**, in this case using the algorithm $\mathrm{R^2U'R'U'RURURU'R}$. To finish, we **return the edges to their original positions** by doing $\mathrm{X}^{-1} = \mathrm{B^2U'B}$. □

Just like with corners, we have some algorithms for solving pairs of 2-cycles.

$$
\begin{aligned}
(13)(24) &= \mathrm{M^2UM^2U^2M^2UM^2} \\
(14)(23) &= \mathrm{UR'U'RU'RURU'R'URUR^2U'R'U}
\end{aligned}
$$

[1]Here, x refers to an entire cube turn. Rotate the entire cube such that the F face becomes the U face and the D face becomes the F face. Also, r refers to a double-layer turn. This means to turn both R and the adjacent middle layer, i.e. $\mathrm{r}^2 = \mathrm{R^2M^2}$.

6.4 Permutation Parity

Sometimes, after all the previous permutation steps, you will be left with one 2-cycle of corners and one 2-cycle of edges. This means there is a permutation parity, in which case this algorithm (or any algorithm that swaps two corners and two edges) must be used.

$$\text{T-perm} = \mathrm{RUR'U'R'FR^2U'R'U'RUR'F'}$$

The T-perm swaps the corners at locations 3 and 4, as well as the edges at locations 2 and 4. As with the other permutation steps, there are restrictions for set-up moves here as well. If you are setting up a corner, F, B, L, and R must be turned in multiples of 180°, as with corner permutation. Likewise, when setting up edges, L and R must be turned in multiples of 180°.

Example. We want to solve the corner cycle (16) and the edge cycle (29). We start by **setting up the pieces** on the U face. We can do this with the set-up move(s) $\mathrm{X} = \mathrm{D'B^2U^2L^2}$. Everything is now set up for out T-perm, so we **permute the pieces** with the algorithm $\mathrm{RUR'U'R'FR^2U'R'U'RUR'F'}$. To finish, we **return the pieces to their original positions** by doing $\mathrm{X}^{-1} = \mathrm{L^2U^2B^2D}$. □

7 Helpful Resources

I took stuff from Shotaro (Macky) Makisumi's website in the making of this guide. His website is cubefreak.net.

Also, there are many other cubers' guides for blindfold cubing, which can be found with a quick Google search!