1\. a) $f(0) = -1 \quad f(2) = -5$

$$m = \frac{f(2) - f(0)}{2-1} = \frac{-5+1}{2} = \boxed{-2}$$

$f(.5) = -2.375 \quad f(1.5) = -5.125$

$$m = \frac{f(1.5) - f(.5)}{1.5-1} = \frac{-5.125 + 2.375}{1} = \boxed{-2.75}$$

$f(.75) = -3.203125 \quad f(1.25) = -4.671875$

$$m = \frac{f(1.25) - f(.75)}{1.25 - .75} = \boxed{-2.9375}$$

$f(.875) = -3.611328125 \quad f(1.125) = -4.357421875$

$$m = \frac{f(1.125) - f(.875)}{1.125 - .875} = \boxed{-2.984375}$$

b) $f'(x) = 3x^2 - 2(2x) - 2 = 3x^2 - 4x - 2$

$f'(1) = 3(1)^2 - 4(1) - 2 = -3$

c) As the ~~xx~~ two points the secant line intersects the function at get closer and closer, the slope of the secant line approaches the derivative at $x = 1$

2. a) $\frac{dy}{dx} = \frac{d}{dx}(4x^3 - 3x^2 + x + 5)$

$= 4\frac{d}{dx}x^3 - 3\frac{d}{dx}x^2 + \frac{d}{dx}x + \frac{d}{dx}5$

$= 4(3x^2) - 3(2x) + 1 + 0$

$= \boxed{12x^2 - 6x + 1}$

b) $\frac{dy}{dx} = \frac{d}{dx}(2x^3 - 5x^2 - 2x + 9)$

$= 2(3x^2) - 5(2x) - 2(1) + 9$

$= \boxed{6x^2 - 10x - 2}$

c) $\frac{dy}{dx} = \sin(2x+1)$

we need to use the chain rule here

$\frac{d}{dx} g(f(x)) = g'(f(x)) * f'(x)$

here $g(x) = \sin(x)$ and $f(x) = 2x+1$

$\frac{d}{dx} = \cos(2x+1) * \frac{d}{dx}(2x+1)$

$= \cos(2x+1) * 2$

$= \boxed{2\cos(2x+1)}$

d) $\frac{dy}{dx} = \frac{d}{dx}\sin(x^2)$

$= \cos(x^2) * \frac{d}{dx}x^2$

$= \boxed{2x\cos(x^2)}$

chain rule again

$g(x) = \sin(x)$ $f(x) = x^2$

e) $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x-1}{2x+5}\right)$ we want to use the quotient rule

$$= \frac{(2x+5)\frac{d}{dx}(3x-1) - (3x-1)\frac{d}{dx}(2x+5)}{(2x+5)^2}$$

$$= \frac{(2x+5)(3) - (3x-1)(2)}{(2x+5)^2}$$

$$= \boxed{\frac{17}{4x^2+20x+25}}$$

f) $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2-x-12}{x+3}\right)$ quotient rule again

$$= \frac{(x+3)\frac{d}{dx}(x^2-x-12) - (x^2-x-12)\frac{d}{dx}(x+3)}{(x+3)^2}$$

$$= \frac{(x+3)(2x-1) - (x^2-x-12)(1)}{(x+3)^2}$$

$$= \frac{2x^2+5x-3-x^2+x+12}{(x+3)^2}$$

$$= \frac{x^2+6x+9}{(x+3)^2} = \boxed{1}$$

or we could have factored $\frac{x^2-x-12}{x+3}$ and simplified before taking the derivative

$$\frac{d}{dx}\left(\frac{x^2-x-12}{x+3}\right) = \frac{d}{dx}\left(\frac{(x-4)(x+3)}{x+3}\right)$$

$$= \frac{d}{dx}x-4$$

$$= \boxed{1}$$

g) $\frac{dy}{dx} = \frac{d}{dx}(\sin(x)\cos(x))$

here we need the product rule

$\frac{d}{dx} f(x)g(x) = f(x)g'(x) + g(x)f'(x)$

$= \sin(x)(-\sin(x)) + \cos(x)\cos(x)$

$= \boxed{\cos^2(x) - \sin^2(x)}$

h) $\frac{dy}{dx} = \frac{d}{dx}\sin^2(x)$

here we can use the chain rule w/ $g(x) = x^2$ and $f(x) = \sin(x)$

$= 2\sin(x) * \frac{d}{dx}(\sin(x))$

$= 2\sin(x)\cos(x)$

or we can use the product rule

$= \frac{d}{dx}(\sin(x)\sin(x))$

$= \sin(x)\cos(x) + \sin(x)\cos(x)$

$= \boxed{2\sin(x)\cos(x)}$

i) $\frac{dy}{dx} = \frac{d}{dx}(\sin(x)/\cos(x))$ quotient rule

$= \frac{\cos(x)\frac{d}{dx}\sin(x) - \sin(x)\frac{d}{dx}\cos(x)}{\cos^2(x)}$

$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$

$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$ trig identity: $\cos^2 x + \sin^2 x = 1$

$= \boxed{\frac{1}{\cos^2 x}}$ remember now: $\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{d}{dx}(\tan(x))$

$\frac{d}{dx}(\tan(x)) = \frac{1}{\cos^2 x}$

j) $\frac{dy}{dx} = \frac{d}{dx} e^{3x}$ chain rule $g(x) = e^x$ $f(x) = 3x$

$= e^{3x} * \frac{d}{dx}(3x)$

$= \boxed{3e^{3x}}$

k) $\frac{dy}{dx} = \frac{d}{dx} e^{2x-1}$ chain rule

$= \left(e^{2x-1}\right) \frac{d}{dx}(2x-1)$

$= \boxed{2e^{2x-1}}$

l) $\frac{dy}{dx} = \frac{d}{dx}\left(2^x\right)$

$= \frac{d}{dx} e^{(\ln 2)x}$ remember $e^{\ln(2)} = 2$ so $e^{\ln(2)x} = 2^x$

$= e^{(\ln 2)x} * \frac{d}{dx} \ln(2)x$ now chain rule

$= \boxed{\ln(2)\, e^{x \ln(2)}}$

m) $\frac{dy}{dx} = \frac{d}{dx}\left(e^{3x}\cos(2x-5)\right)$ product rule

$= e^{3x} \frac{d}{dx}\cos(2x-5) + \cos(2x-5)\frac{d}{dx}e^{3x}$ chain rule

$= e^{3x}\left(-2\sin(2x-5)\right) + \cos(2x-5)\left(3e^{3x}\right)$

$= -2e^{3x}\sin(2x-5) + 3e^{3x}\cos(2x-5)$

n) $\frac{dy}{dx} = \frac{d}{dx} \ln(x^2)$

chain rule
$g(x) = \ln(x) \quad f(x) = x^2$

$= \frac{1}{x^2} * \frac{d}{dx} x^2$

$= \frac{1}{x^2} * 2x$

$= \boxed{\frac{2}{x}}$

or we could have used $\ln(x^2) = 2\ln(x)$

$\cancel{\frac{d}{dx}} = \frac{d}{dx} 2\ln x$

$= \boxed{2/x}$

o) $\frac{dy}{dx} = \frac{d}{dx}(\log_{10}(x))$

$\log_{10} x = \frac{\ln x}{\ln 10}$
exponent change of base rule

$= \frac{d}{dx} \frac{\ln x}{\ln 10}$

$= \boxed{\frac{1}{x \ln 10}}$

p) $\frac{dy}{dx} = \ln(\sin((x+2)^2))$ lots of chain rules

$= \frac{1}{\sin((x+2)^2)} * \frac{d}{dx} \sin((x+2)^2)$

$= \frac{1}{\sin(x+2)^2} * \cos((x+2)^2) * \frac{d}{dx}(x+2)^2$

$= \frac{\cos(x+2)^2}{\sin(x+2)^2} * 2x+4$

$= \boxed{\frac{2x+4}{\tan(x+2)^2}}$

$\frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$

2^{nd} Derivatives for problem 2

2. a) $24x - 6$

b) $12x - 10$

c) $-4\sin(2x+1)$

d) $-4x^2 \sin(x^2) + 2\cos(x^2)$

e) $\dfrac{-68}{(2x+5)^3}$

f) 0

g) $-4\sin(x)\cos(x)$

h) $2\cos^2(x) - 2\sin^2(x)$

i) $2\sin(x) / \cos^3(x)$

j) $9e^{3x}$

k) $4e^{2x-1}$

l) $\ln^2 2 \; 2^x$

m) $-4e^{3x}\cos(2x-5)$

n) $-\dfrac{2}{x^2}$

o) $-\dfrac{1}{x^2 \ln 10}$

p) $\dfrac{-4x^2 - 16x - 16 + \sin(2(x+2)^2)}{\sin^2((x+2)^2)}$

3. a) $f'(x) = 12x^3 - 12x^2 - 24x$

$f''(x) = 36x^2 - 24x - 24$

b) $f'(x) = 0$

$0 = 12x^3 - 12x^2 - 24x$

$0 = 12(x)(x-2)(x+1)$

$x = 0, \quad x = 2, \quad x = -1$ are the zeroes

These points correspond to the maxima and minima of the graph of $f(x)$
These points are called critical points

c) $f''(0) = -24 \rightarrow$ maxima

$f''(-1) = 36 \rightarrow$ minima

$f''(2) = 72 \rightarrow$ minima

At a critical point, if the second derivative $f''(x)$ is negative the graph is at a maxima.
If $f''(x)$ is positive, the graph is at a minima.

4. a) $\int 3x^2 + 4x + 1 \, dx$

$= \int 3x^2 \, dx + \int 4x \, dx + \int 1 \, dx$

$= \boxed{x^3 + 2x^2 + x + C}$

b) $\int x^4 + 4x^3 - 3x + 7 \, dx$

$= \int x^4 dx + \int 4x^3 dx - \int 3x \, dx + \int 7 \, dx$

$= \boxed{\frac{1}{5}x^5 + x^4 - \frac{3}{2}x^2 + 7x + C}$

c) $\int e^{2x} \, dx$

$= \boxed{\frac{1}{2}e^{2x} + C}$

d) $\int \frac{1}{x+2} \, dx$

$= \boxed{\ln(x+2) + C}$

You can check these ~~and you wi~~ by taking the first derivative to get back the original function

C is the constant of integration

5. a) $\int_0^5 6x^2+2x-15\,dx$

$= 2x^3+x^2-15x \Big|_0^5$

$= 2(5)^3+(5)^2-15(5)-\left[2(0)^3+(0)^2-15(0)\right]$

$= \boxed{200}$

b) $\int_{-2}^{2} x^4+4x^3-3x+7\,dx$

$= \frac{1}{5}x^5+x^4-\frac{3}{2}x^2+7x \Big|_{-2}^{2}$

$= \frac{1}{5}(2)^5+(2)^4-\frac{3}{2}(2)^2+7(2)-\left[\frac{1}{5}(-2)^5+(-2)^4-\frac{3}{2}(-2)^2+7(-2)\right]$

$= \frac{152}{5}-\left(-\frac{52}{5}\right) = \boxed{\frac{204}{5}}$

c) $\int_{-\pi}^{\pi} \sin(x)\,dx$

$= -\cos(x) \Big|_{-\pi}^{\pi}$

$= -\cos(\pi)-\left[-\cos(-\pi)\right] = 1-1 = \boxed{0}$

d) $\int_{-\pi}^{\pi} \cos(x)\,dx$

$= \sin(x) \Big|_{-\pi}^{\pi}$

$= \sin(\pi)-\sin(-\pi) = 0-0 = \boxed{0}$

6. a) $|v| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

$\theta = \tan^{-1}(4/3) \approx 53.13°$

b) $|v| = \sqrt{2^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$

$\theta = \tan^{-1}(-2/2) = -45°$

c) $x = 3\sqrt{2}\cos(135°) = 3\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -3$

$y = 3\sqrt{2}\sin(135°) = 3\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 3$ $\boxed{(-3,3)}$

d) $x = \frac{4\sqrt{3}}{3}\cos(30°) = \left(\frac{4\sqrt{3}}{3}\right)\left(\frac{\sqrt{3}}{2}\right) = 2$

$y = \frac{4\sqrt{3}}{3}\sin(30°) = \frac{4\sqrt{3}}{3}\left(\frac{1}{2}\right) = \frac{2\sqrt{3}}{3}$ $\boxed{\left(2, \frac{2\sqrt{3}}{3}\right)}$

7. a) $3\sqrt{2} \measuredangle 45° = (-3,3)$

$(-3,3) + (3,6\rangle = \langle 0,9\rangle = \boxed{9 \measuredangle 90°}$

b) $\frac{4\sqrt{3}}{3} \measuredangle 30° = \left\langle 2, \frac{2\sqrt{3}}{3}\right\rangle$

$\frac{4}{3} \measuredangle -60° = \left\langle \frac{2}{3}, -\frac{2\sqrt{3}}{3}\right\rangle$

$\left\langle 2, \frac{2\sqrt{3}}{3}\right\rangle + \left\langle \frac{2}{3}, -\frac{2\sqrt{3}}{3}\right\rangle = \left(\frac{8}{3}, 0\right) = \boxed{\frac{8}{3} \measuredangle 0°}$