1 Thermodynamics: Equations and constants

- Energy, speed, wavelength and frequency
  - Speed of light $c = 299,792,458$ meters/second $= 3 \cdot 10^8 m/s$
  - Wavelength λ, frequency ν:
    $\Rightarrow \quad c = \lambda \cdot \nu$
  - Energy E, Planck's constant $h = 6.63 \cdot 10^{-34}$ Joule-seconds
    $\Rightarrow \quad E = h \cdot \nu$.
    Example: The energy of a single photon in a microwave (wavelength 10^{-2} meters, frequency $3 \cdot 10^{10}$ Hertz) is
    $$E = h \cdot \nu = (6.63 \cdot 10^{-34}) \cdot (3 \cdot 10^{10}) = 2 \cdot 10^{-23}\, J$$
    a very, very small amount of energy. Kitchen microwaves have to put out billions of billions of photons to be able to heat up your food.

- Planck's law of radiation: The intensity of radiation with frequency ν coming from an object of temperature T is
  $$I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$
  where $k = 1.38 \cdot 10^{-23}$ Joules per Kelvin is the Boltzmann constant.

- Wien's displacement law: The wavelength of the maximum amount of radiation is
  $$\lambda_{max} = \frac{b}{T}$$
  where $b = 2.9$ millimeters-Kelvin is another constant.

  Example: The sun emits light in the visible part of the spectrum, with a wavelength around 502 nanometers, or .0005 millimeters. From Wien's law, the temperature of the sun is about
  $$T = b/\lambda_{max} = 2.9/.0005 = 5777\, K$$
  Example 2: Mammals have body temperatures around 310 Kelvin (37 degrees Celsius, 98.6 degrees Fahrenheit). From Wien's law, most of the radiation coming off of us has wavelength
  $$\lambda_{max} = b/T = 2.9/310 = .00936\, mm$$
  or about 10^{-5} meters. This corresponds to infrared radiation.

- Stefan-Boltzmann law: The *total* amount of power radiated by an object at temperature T is
  $$P = \epsilon\sigma \cdot T^4$$
  where $\sigma = 5.67 \cdot 10^{-8}$ watts per square meter per K^4 and ϵ is called the *emissivity* of the object.

  Example: If the sun were a perfect radiator ($\epsilon = 1$), then
  $$P = \sigma \cdot T^4 = (5.67 \cdot 10^{-8}) \cdot (5777)^4 = 6.3 \cdot 10^7$$

is the power output per square meter. The radius of the sun is about 696,000 kilometers, so its surface area is $6.1 \cdot 10^12$ square kilometers. The total energy output of the sun is

$$3.8 \cdot 10^{26} \text{ Watts.}$$

This is about 25 trillion times the total amount of power consumed worldwide ($474 \cdot 10^{18}$ Joules per year, or $2.53 \cdot 10^{13}$ Watts).

Example 2: The average temperature of the surface of the earth is about 15 degrees Celsius (59 degrees Fahrenheit), or 288 Kelvin. The earth is not a perfect radiator, but instead has an emissivity of $\epsilon = .6$. The power output is

$$P = (5.67 \cdot 10^{-8}) \cdot (288)^4 = 390$$

Watts per square meter. (Recall that the incoming energy is on average 250 Watts per square meter.) The earth has a surface area of $5 \cdot 10^8$ square kilometers, so the total output of the earth is

$$2 \cdot 10^{17} \text{ Watts}$$

or 8,000 times the worldwide power consumption.