Please write legibly and neatly. This problem set is due Saturday, February 13. NOTE: If you want extra problems for extra practice, I will also be happy to provide those.

Problem 1. Read the Pre-Algebra notes, which are available on the HSSP Course Catalogue on the ESP MIT Website. By "read," I mean that you should look over this document to review the material presented in the first lecture until you are reasonably comfortable with things like sets, maps, unions, intersections, etc. If you have any questions, feel free to contact me by email.

Problem 2. Here we consider some set theory under mappings.

(a) For a function $f : X \to Y$ for some sets X, Y, and any subset $C \subset X$, we denote $f(C)$ as the set of all members of Y such that there exists a $c \in C$ for which $f(c) \in Y$. Or, equivalently, we can define it as the set that has as its members all elements $f(c)$ for each $c \in C$. (If you can't understand this definition on the first try, read it again slowly and remember all the definitions we brought up in class!)

Consider the mapping $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |5x|$. Describe the set $f(\mathbb{Z})$ (Recall that $\mathbb{R}$ denotes the real numbers and $\mathbb{Z}$ denotes the integers).

(b) Consider a set X, and two subsets $A, B \subset X$. Show that $f(A \cup B) = f(A) \cup f(B)$.

(HINT: To show two sets A and B are equal, remember that you must show that $A \subset B$ and $B \subset A$).

(c) Is it true that $f(A \cap B) = f(A) \cap f(B)$? If yes, prove it. If not, explain why (usually through giving an explicit example in which this statement is not true).

Problem 3. Here, we will prove some basic properties of compositions of mappings. If you're not sure about how to proceed, draw a simple diagram to convince yourself first!

(a) Show that if $f : X \to Y$ and $g : Y \to Z$ are both surjective functions, then $g \circ f : X \to Z$ is also surjective.

(b) Show that if $f : X \to Y$ and $g : Y \to Z$ are both injective functions, then $g \circ f : X \to Z$ is also injective.

(c) Conclude that if $f : X \to Y$ and $g : Y \to Z$ are both bijective functions, and hence invertible, then $g \circ f : X \to Z$ is also bijective, and hence invertible.

Problem 4. Using induction, show that

$$7^n - 6n - 1$$

is divisible by 36 for all positive integers n.