Vectors and Beyond: Day 2 Notes

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1 In-Class Problems

1. What is the set of all vectors whose dot product with $(2, 2, 1)$ is 9?
2. What is the set of all vectors whose cross product with $(1, 0, 0)$ has magnitude 1?

2 Notes

2.1 Column Vector Notation

At first this is nothing but a notation convention. From now on we'll represent vectors as vertical arrays of numbers, such as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

2.2 Products of Vectors

The in-class problems led naturally to the equation of a plane, one of the first applications we've seen of the dot product:

$$\mathbf{v} \cdot \mathbf{r} = c$$

...where:

- $\mathbf{v}$ is a vector perpendicular to the plane
- $\mathbf{r}$ is the column vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ containing our variables
- c is some constant that controls how far away the plane is from the origin

2.3 Matrices

An $m \times n$ *matrix* is written as a grid of numbers with m rows and n columns. We focused initially on 2×2 matrices, thinking of them as *linear transformations*. That is, we can multiply a vector $\mathbf{v}$ by a matrix A, and we'll get back another vector. If we have two vectors $\mathbf{u}$ and $\mathbf{v}$, then the following holds:

$$A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$$