M15936: Set theory and applications

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Class Description

This class is all about mathematical objects called sets. A set is a container that can hold anything, like numbers, shapes, graphs, and even other sets. After introducing basic set operations and their symbols, we will tour how set theory is used in two important areas: probability theory and computer logic. At the end of the course, we will discuss the role set theory has in more abstract mathematical areas. Part of each class will be dedicated to solving example problems of varying difficulty.

Syllabus

Week I. Set theory background and motivation. Subsets and supersets

Week II. Set operations and quantifiers

Week III. Set operations and quantifiers continued, set theory of computer logic, truth tables, logic gates

Week IV. Probability theory

Week V. Discussion of set theory in formal mathematics, overflow day

Course outline

Part I: Set theory fundamentals

1. What is generality? Why it's important in mathematics.
2. Small note on definitions vs. equality; $:=$ vs. $=$.
3. Define and discuss sets, with examples.
   (a) $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
   (b) $\mathbb{C}$, $\mathbb{S}$
4. Objects and their operations in math. Brainstorm. What are some operations we do to numbers? How about functions?
5. The empty set. The symbol $\in$.
6. Implicit vs. explicit set definitions. Sets containing other sets: subsets and supersets. Set equality.
7. Discussion: what are some ways we could operate with sets? Venn diagrams.
8. Set operations: equality, union, intersection, complement, difference, Cartesian product, count measure $\#$.
9. Indexed families of sets. Partitions.

Part II: Applications

1. Set theory for computer logic
2. The quantifiers $\forall$ and $\exists$
3. Truth tables and logic gates as their physical realization
4. Set theory for probability theory
5. Events as sets
6. Probability measures