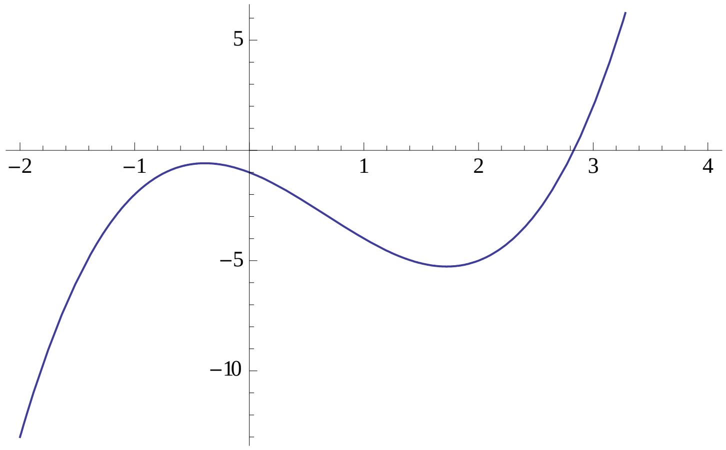
Calculus and Vectors Worksheet

1. The derivative of a function at a point can be thought of as the slope of the line tangent to the function at that point. Slope is the change in f(x) per change in x: $m = \frac{\Delta f(x)}{\Delta x}$. To get a sense of this let's look at the function $f(x) = x^3 - 2x^2 - 2x - 1$ pictured below:



We want to estimate the derivative at $x = 1$ and we'll do that by calculating the slopes of different secant lines (lines that intersect the function at 2 points).

a. Calculate the slope of the line that runs through the points:
   i. $(0,\ f(0))$ and $(2,\ f(2))$
   ii. $(.5, f(.5))$ and $(1.5, (f(1.5))$
   iii. $(.75, f(.75))$ and $(1.25, f(1.25))$
   iv. $(.875, f(.825))$ and $(1.125, f(1.125))$

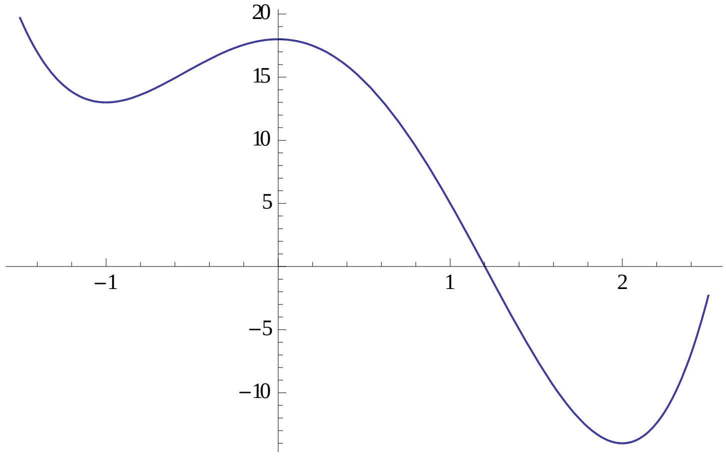
b. Now calculate the first derivative $f'(x)$ and evaluate at $x = 1$.

c. How does the derivative at $x = 1$ compare to the slopes of the secant lines?

2. Find the first derivative dy/dx of the following functions. You can find the second derivative d^2y/dx^2 if you want a challenge.

a. $y = 4x^3 - 3x^2 + x + 5$
b. $y = 2x^3 - 5x^2 - 2x + 9$
c. $y = \sin(2x + 1)$
d. $y = \sin(x^2)$
e. $y = (3x - 1)/(2x + 5)$
f. $y = (x^2 - x - 12)/(x + 3)$
g. $y = \sin(x)\cos(x)$
h. $y = \sin^2(x)$
i. $y = \sin(x) / \cos(x)$
j. $y = e^{3x}$
k. $y = e^{2x-1}$
l. $y = 2^x$
m. $y = e^{3x}\cos(2x - 5)$
n. $y = \ln(x^2)$
o. $y = \log_{10}(x)$
p. $y = \ln(\sin((x + 2)^2)$

3. The first and second derivatives are useful for finding the minima and maxima of functions. Let's look at this with the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 18$ pictured below.



   a. Find the first derivative $f'(x)$ and second derivative $f''(x)$.
   b. Find the zeroes of the first derivative. For what values of x is $f'(x)$ = 0? Look at the graph of $f(x)$. What is special about these points?
   c. Evaluate the second derivative $f''(x)$ at the zeroes of the first derivative. How does the sign of the second derivative correspond with the graph?

4. Take the indefinite integral by finding the antiderivative.
   a. $\int 3x^2 + 4x + 1\, dx$
   b. $\int x^4 + 4x^3 - 3x + 7\, dx$
   c. $\int e^{2x}\, dx$
   d. $\int 1/(x + 2)\, dx$

5. Take the definite integral.
   a. $\int_0^5 6x^2 + 2x - 15\, dx$
   b. $\int_{-2}^{2} x^4 + 4x^3 - 3x + 7\, dx$
   c. $\int_{-\pi}^{\pi} \sin(x)\, dx$
   d. $\int_{-\pi}^{\pi} \cos(x)\, dx$

6. Convert these vectors from rectangular form to polar form or vice versa.
   a. $V = (3, 4)$
   b. $V = (2, -2)$
   c. $|V| = 3\sqrt{2}, \theta = 135°$
   d. $|V| = 4\sqrt{3} / 3, \theta = 30°$

7. Add these vectors by converting to rectangular form. Give answers in polar form.
   a. $|V| = 3\sqrt{2}, \theta = 45°$ and (3,6)
   b. $|V| = 4\sqrt{3} / 3, \theta = 30°$ and $|V| = 4 / 3, \theta = -60°$