

Goals: This course gives the student an introduction into abstract algebra. Abstract algebra is a fundamentally important subject in theoretical mathematics, and this class is intended to make the young mathematician comfortable in the language and concepts of this crucial branch of mathematics. The first two major parts of the class will consist of group theory and linear algebra, and time permitting, we will continue to develop concepts in ring theory. An emphasis will be placed on learning how to construct rigorous proofs.

Prerequisites: You should be familiar with math through algebra 2. This means understanding how to manipulate algebraic expressions and how to deal with functions, matrices, and complex numbers. The beautiful thing about abstract algebra is that it is so fundamental that very little prior knowledge is needed; however, you should still know enough of the basics to be able to comfortably follow a lecture.

Lectures: The class will be conducted in two-hour weekly lectures. Class participation and questions are encouraged, and there will be at least one short break in the middle of each lecture for the students to work on short problems. The material is fairly cumulative, so all lectures should be attended to maximize understanding.

Homework: It is impossible to learn math without doing math on your own, and hence weekly problem sets will be assigned. Although no ESP homework is technically mandatory, it is highly important that you try these assignments. The assignments will be relatively short (i.e. 1-3 hours) as to be more manageable.

Textbook: Although there is no official textbook for the class, an excellent introduction to abstract algebra can be found in Artin's Algebra. Lectures will be self-contained, but any student who wants a good textbook for supplementary study should try this text.

Weekly Schedule (SUBJECT TO CHANGE)

Week 1: Introduction. Sets and maps between sets. Proofs and proofwriting. Logical negation and De Morgan's Laws. Induction, contrapositive. Groups and examples of groups. General Linear Group and symmetric groups.

Week 2: Subgroups, cyclic groups, generators. Group homomorphisms and isomorphisms. Kernel and Normal Subgroups. Direct Product of groups.

Week 3: Equivalence classes and cosets. The Lagrange Counting Formula. Quotient groups. Isomorphism Theorems.

Week 4: Group actions. Orbit-Stabilizer Theorem. Sylow Theorems. Cayley's Theorem. Free Groups and Presentations.

Week 5: Vector spaces. Linear transformations and matrices. Bases and change of basis. Dimension. Rank Nullity Theorem.

Week 6: Linear operators. Eigenvalues and eigenvectors. Characteristic polynomial. Matrix diagonalization.

Week 7: Inner product spaces and orthogonality. Bilinear forms. Spectral theorem.

Week 8 and beyond: Other topics. Rings, fields, modules, etc.